

Math 302
Homework 6

1. Find the inverse function of

$$f(x) = \frac{3x+2}{x-1}$$

Express it in a way as simple as possible. Then do the same for a general case:

$$f(x) = \frac{ax+b}{cx+d}$$

where a, b, c and d are some constants.

2. Count how many surjective and how many injective functions are there from A to B

- (i) if A is a two-element set and B is a three-element set
(ii) if A is a three-element set and B is a two-element set

3. How many functions is there from an n -element set to an m -element set?

4. Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are both surjective, then so is $g \circ f$.

5. Define functions f and $g: \mathbf{R} \rightarrow \mathbf{R}$ by $f(x) = x^2$ and $g(x) = x^2 - 1$.

(a) Find functions: $f \circ f, f \circ g, g \circ f, g \circ g$

(b) Find the elements of set $\{x \in \mathbf{R} \mid f \circ g(x) = g \circ f(x)\}$

6. For each of the following functions determine whether it is surjective, injective, bijective, or neither:

(a) $f: \mathbf{R} \rightarrow \mathbf{R}, f(x) = 2^{-x} + 1$

(b) $f: [1, \infty) \rightarrow \mathbf{R}, f(x) = x^2 + 1$

(c) $f: \mathbf{R} \rightarrow \mathbf{R}, f(x) = \sin(x)$

(d) $f: \mathbf{N} \rightarrow \mathbf{N}, f(n) =$ the smallest divisor of n greater than 1

7. Give an example of a continuous function $f: (0, \infty) \rightarrow [0, \infty)$ that is surjective.

8. Let $f: \mathbf{R} \rightarrow \mathbf{R}$. Denote $f^{-1}(A) = \{x \in \mathbf{R} \mid f(x) \in A\}$. Prove that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.