

# Introduction to Real Analysis

Math 452 (Jerzy Kocik)

## Topics

### 0. Preparation

propositional logic, set theory (Russell paradox), predicative logic, metric spaces, functions, relations, equivalence classes, proofs

### 1. What are the Real Numbers

irrationality of  $\sqrt{2}$ , supremum, the Axiom of Completeness, Archimedean property, cardinality, Cantor's Theorem.

### 2. Sequences and Series

Rearrangements of Infinite Series, algebraic and order limit theorems, monotone convergence theorem, Bolzano-Weierstrass theorem, Cauchy Criterion, properties of infinite series, double summations and products of infinite series.

### 3. Basic Topology of $\mathbf{R}$

Set: open, closed, compact, connected, dense. Completion. Cantor set. Baire's Theorem.

### 4. Limits and Continuity

functions (examples: Dirichlet, Thomae, etc), continuity and compactness, Intermediate Value Theorem, Sets of Discontinuity.

### 5. The Derivative

Intermediate Value Property, Mean Value Thrm, Darboux Thrm, etc, examples of a continuous nowhere-differentiable functions.

### 6. Sequences and Series of Functions:

uniform convergence of sequences of functions, differentiation, power Series (Taylor Series).

### 7. The Riemann Integral

definition via Darboux sums, discontinuities, various properties, Fundamental Theorem of Calculus, Lebesgue's criterion for Riemann integrability.

### 8. Additional Topics

Various constructions of  $\mathbf{R}$  from  $\mathbf{Q}$ .