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Markov Random Fields with Stable Points for Region Labeling

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ABSTRACT

A technique that improves both the convergence rate and the final results for Markov random fields in the context of region labeling problems is described. Stable points are introduced via a deterministic procedure at the outset of the algorithm and these are used to initiate the labeling process. The stable points provide contextual information by inducing pockets of "certainty" in the stochastic evolution of the algorithm. A discussion of the precise mathematical effect of the stable points on the random field is included.

1.INTRODUCTION

The human visual system is an extraordinary structure which performs complex calculations with both speed and sensitivity. There is still a great deal of debate concerning the nature of the computations involved in human vision, but agreement has been reached on certain aspects of the problem. It seems clear that vision is a hierarchical process with certain tasks such as edge detection performed almost immediately in a "pre-attentive" fashion, perhaps even at the retinal level.

Other conclusions can be drawn from the structure of the process. The vision system is essentially a series of labeling operations; each one more complex than the previous. A given level of the hierarchy uses the output from the previous level and additional contextual information from the image to form the next stage output. Finally, the speed at which the initial pre-attentive tasks are completed point to a fundamentally parallel algorithm.

The simple (and, hopefully, not controversial) inferences from the previous paragraph lead one to suggest a hierarchical Markov random field (MRF) as a mathematical model that captures the essential properties of the vision system. MRF algorithms are spatially parallel by their very nature. Their design is flexible enough to easily incorporate the necessary hierarchical structure. The inclusion of additional contextual information at each level can be modeled in very natural manner within the MRF design (Budzban and Stirewalt, 1994).

Historically, the problem with MRF algorithms has been the convergence rate of the stochastic optimization strategy, simulated annealing. This difficulty is intensified by the need for multiple levels of MRF's to model the entire vision system. An additional problem occurs in complex real images, as opposed to simplistic computer generated pictures. No matter how elementary the form of the chosen potential function, The associated probability distribution will be extremely multi-modal. While some of these minimum energy configurations will correspond to reasonable labelings of the image, many will seem to have little correspondence. The convergence of the MRF to one of these minimum energy configurations is assured by the structure of the annealing algorithm, but the possibility for unstable transitions between minima is unsettling.

Introducing stable points in the labeling array drastically improves both the convergence rate of the MRF and the instability in its labeling. The stable points are introduced at the outset of the algorithm in the initial configuration and remain unchanged during the temporal evolution of the random field. The stable points provide sources of certainty and contextual information.

The structure of the paper will be as follows. After this introduction, a discussion of the procedure and an analysis of the stable points in the context of traditional Markov chain terminology will appear in Section 2. Section 3 will contain some final comments.

2. PROCEDURE

As is now well known, a MRF is the natural spatially indexed generalization of a one-dimensional time-indexed Markov chain. That is, if X_s is the random variable associated with the site s, then

$$P(\ X_s=x_s \mid X_r=x_r\ ,\ r\neq s\ ) = P(\ X_s=x_s \mid X_r=x_r\ ,\ r\in G_s\ )$$

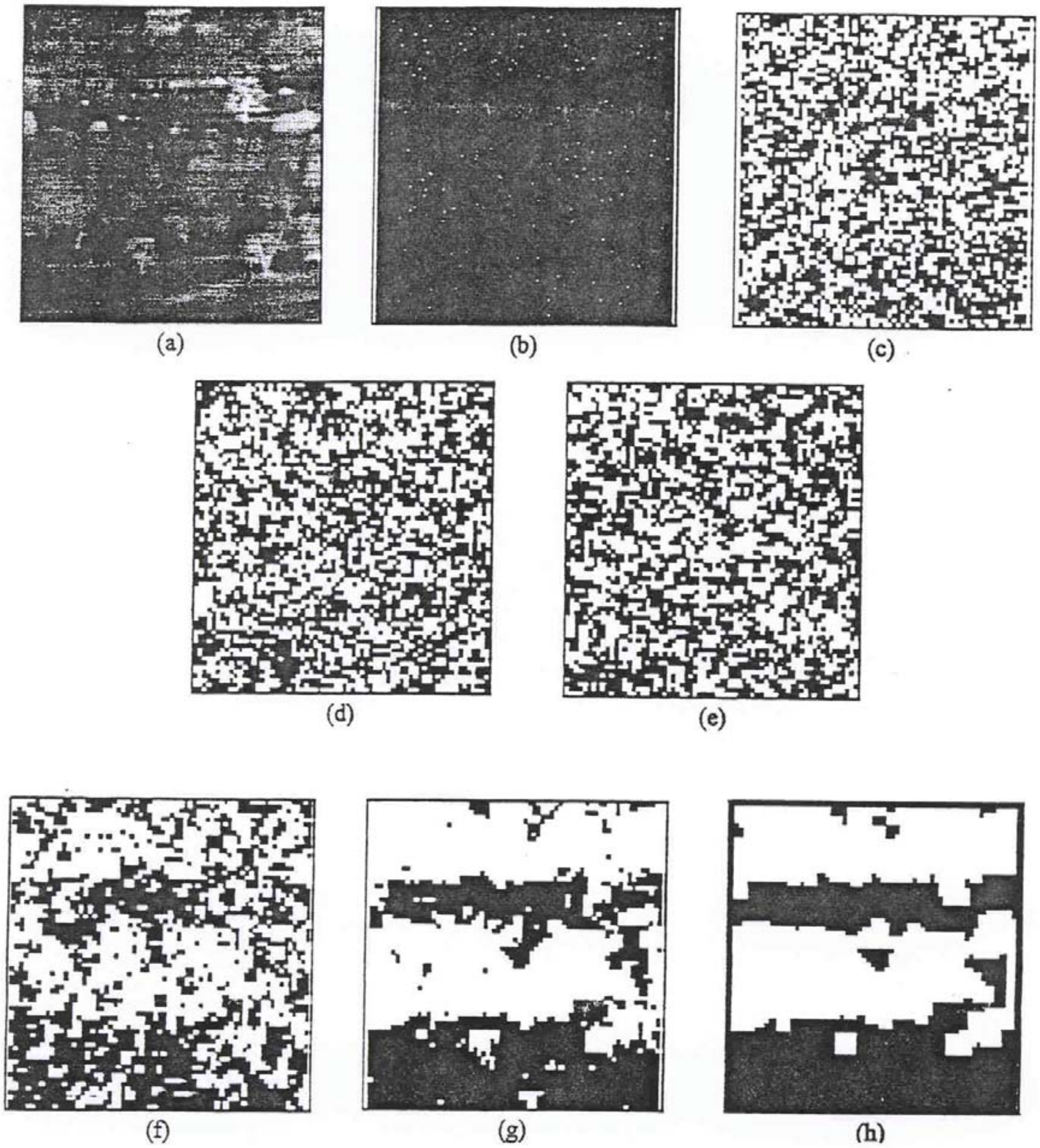
where G_s is the set of spatial neighbors of the site s. Every MRF has an equivalent Gibbs formulation for some potential function U which allows for simple computation of the local probabilities. That is,

$$Prob(\omega) = \frac{1}{Z}\exp(\ \frac{-U(\omega)}{T}\ )$$

where Z is the normalizing constant and T can be thought of as the temperature of the system. See (Geman and Geman, 1984) for the relevant details.

Now G_s is spatially homogeneous with respect to s and since the potential function U is a simple sum of local interactions, the same computations occur at each pixel site. The only problem with immediate parallel implementation is that for neighboring sites s, t their neighborhood structures G_s and G_t intersect. Thus, due to the effect of the conditional probabilities, updating the labels for neighboring sites cannot be done at the same time. There have been several descriptions in the literature of parallel algorithms for MRF's (Murray, et al., 1986; Budzban and DeCatrel, 1991).

To initiate the simulated annealing algorithm, a standard statistical pixel classifier is used to label the points and then a confidence measure is associated with each pixel label. This confidence measure can be as simple as the inverse of the distance of the feature vector for that pixel to the centroid of the pattern cluster for which the pixel was labeled. Then a small



(a) (b) (c) (d) (e) (f) (g) (h)

Figure 1. a) original image b) stable points c) initial label configuration d) after 100 iterations e) after 200 iterations f) after 300 iterations g) after 350 iterations h) after 400 iterations with one dilate and erode.

percentage (less than 1%) of the most confident label are chosen and fixed throughout the process. The rest of the pixels are labeled entirely at random and this labeling array is used to initiate the evolution of the process. (See Figure 1.)

Simulated annealing is most precisely viewed as a non-stationary Markov chain with the possible labeling configurations as the states of the chain. It induces a temporal parameter, say k, so that the configuration at time k is $X(k) = (X_s(k))$. Two different configurations are said to communicate if there is a positive probability that the Markov chain can transform one into the other (and visa versa) in a finite number of steps. That is, for some $m > 0$

$$P(\ X(k+m) = w_1 \mid X(k) = w_0\ ) > 0 \quad ,where\ w_1 \neq w_0.$$

Thus, X(k) has a single ergodic communication class and is said to be irreducible.

Now let $S = \{ s_1, s_2, \ldots, s_j \}$ be the set of stable sites and let w_0, w_1 be configurations which have different labels at some site, say s_j. Then for all $m > 0$,

$$P(\ X(k+m) = w_1 \mid X(k) = w_0\ ) = 0$$

since no updates occur at site s_j. Thus, X(k) is a non-stationary reducible Markov chain with h^k ergodic components, where h is the number of possible labels at each site and k the number of stable sites.

3. FINAL COMMENTS

There are a great deal of interesting theoretical questions concerning both the dynamics and asymptotics of the non-stationary reducible chains that are induced by the introduction of the stable points. In terms of applications, however, the goal is clear. One should choose the minimum number of stable points so that the reasonable labelings of the original image are uniquely represented in the ergodic components that are produced. One seeks to ensure that there is at most one energy minima in each of the components. More precisely, the wish is that the conditional distribution induced by the chosen initial labeling converges almost surely to the one most probable (hopefully reasonable) labeling within the communication class. This is, in fact, our conjecture.

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