Math 107
Spring 2011
Worksheet 7

Addition of Rational Expressions/
Operations with radicals

Name Answer Key
Date______________
Section______________

Simplify. Write each rational expression in lowest terms. Show all work.

1. $\frac{7}{6}-\frac{5}{12}$ LCD=12

$\left(\frac{2}{2}\right)\frac{7}{6}-\frac{5}{12}=\frac{14}{12}-\frac{5}{12}$
$=\frac{9}{12}=\boxed{\frac{3}{4}}$

2. $\frac{11}{18}-\left(\frac{-3}{4}\right)+2$ LCD=36

$=\left(\frac{2}{2}\right)\frac{11}{18}-\left(\frac{-3}{4}\right)\left(\frac{9}{9}\right)+\frac{2}{1}\left(\frac{36}{36}\right)$
$=\frac{22}{36}-\left(\frac{-27}{36}\right)+\frac{72}{36}$
$=\frac{22+27+72}{36}=\boxed{\frac{121}{36}}$

3. $\frac{3x}{x^2-9}-\frac{2x+3}{x^2-9}=\frac{3x-(2x+3)}{x^2-9}=\frac{3x-2x-3}{x^2-9}=\frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)}=\boxed{\frac{1}{x+3}}$

Assume that the expressions given are denominators of fractions. Find the least common denominator (LCD) for each group.

4. $24a^3b^4$, $18a^5b^2$

24: 24, 48, 72, 96, 120
18: 18, 36, 54, 72, 90

a: a^3, a^5 Greatest Exponent a^5

b: b^4, b^2 Greatest Exponent b^4

LCD=$\boxed{72a^5b^4}$

5. m^2+2m-3, m^2+5m+6

$(m+3)(m-1)$ $(m+3)(m+2)$

LCD=$\boxed{(m+3)(m-1)(m+2)}$

Steps for Adding or Subtracting Rational Expressions with different denominators

Step 1: Factor each denominator.

Step 2: Find LCD.

Step 3: Write all rational expressions with this LCD.

Step 4: Add or Subtract numerators placing result over LCD.

Step 5: Write answer in lowest terms.

Simplify. Write each rational expression in lowest terms. Show all work.

6. $\frac{9}{x-2}-\frac{3}{x}$ LCD: $x(x-2)$

$$\left(\frac{x}{x}\right)\frac{9}{x-2}-\frac{3}{x}\left(\frac{x-2}{x-2}\right)=\frac{9x}{x(x-2)}-\frac{3(x-2)}{x(x-2)}=\frac{9x-3(x-2)}{x(x-2)}$$

$$=\frac{9x-3x+6}{x(x-2)}=\frac{6x+6}{x(x-2)}=\boxed{\frac{6(x+1)}{x(x-2)}}$$

7. $\frac{3}{x^2+4x+4}+\frac{7}{x^2+5x+6}$ LCD: $(x+3)(x+2)^2$

Factor: $(x+2)(x+2)$ $(x+3)(x+2)$

$= (x+2)^2$

$$=\frac{3}{(x+2)^2}+\frac{7}{(x+2)(x+3)}$$

$$=\left(\frac{x+3}{x+3}\right)\frac{3}{(x+2)^2}+\frac{7}{(x+2)(x+3)}\left(\frac{x+2}{x+2}\right)$$

$$=\frac{3(x+3)}{(x+2)^2(x+3)}+\frac{7(x+2)}{(x+2)^2(x+3)}=\frac{3x+9+7x+14}{(x+2)^2(x+3)}=\boxed{\frac{10x+23}{(x+2)^2(x+3)}}$$

8. $\frac{5x}{x+3}+\frac{x+2}{x}-\frac{6}{x^2+3x}$ $x(x+3)$ Factor LCD: $x(x+3)$

$$=\frac{5x}{x+3}+\frac{x+2}{x}-\frac{6}{x(x+3)}=\left(\frac{x}{x}\right)\frac{5x}{x+3}+\left(\frac{x+3}{x+3}\right)\frac{x+2}{x}-\frac{6}{x(x+3)}$$

$$=\frac{5x^2}{x(x+3)}+\frac{(x+3)(x+2)}{x(x+3)}-\frac{6}{x(x+3)}=\frac{5x^2+x^2+5x+6-6}{x(x+3)}=\frac{6x^2+5x}{x(x+3)}$$

$$=\frac{\cancel{x}(6x+5)}{\cancel{x}(x+3)}=\boxed{\frac{6x+5}{x+3}}$$

9. $\frac{2x}{x^2-1}+\frac{-1}{x+1}$ LCD: $(x-1)(x+1)$

Factor $(x-1)(x+1)$

$$=\frac{2x}{(x-1)(x+1)}+\frac{-1}{x+1}\left(\frac{x-1}{x-1}\right)=\frac{2x}{(x-1)(x+1)}+\frac{-1(x-1)}{(x+1)(x-1)}=\frac{2x-x+1}{(x+1)(x-1)}$$

$$=\frac{\cancel{x+1}}{(x-1)\cancel{(x+1)}}=\boxed{\frac{1}{x-1}}$$

Perform the indicated operations and simplify.

10. $2\sqrt{12}+\sqrt{48}-\sqrt{3}$

$2\sqrt{4\cdot3}+\sqrt{16\cdot3}-\sqrt{3}$

$=2\sqrt{4}\sqrt{3}+\sqrt{16}\sqrt{3}-\sqrt{3}$

$=2\cdot2\sqrt{3}+4\sqrt{3}-\sqrt{3}=4\sqrt{3}+4\sqrt{3}-\sqrt{3}$

$=8\sqrt{3}-\sqrt{3}=\boxed{7\sqrt{3}}$

11. $3\sqrt{5}-7\sqrt{45}+\sqrt{12}$

$3\sqrt{5}-7\sqrt{9\cdot5}+\sqrt{4\cdot3}$

$=3\sqrt{5}-7\sqrt{9}\sqrt{5}+\sqrt{4}\sqrt{3}$

$=3\sqrt{5}-7\cdot3\sqrt{5}+2\sqrt{3}$

$=3\sqrt{5}-21\sqrt{5}+2\sqrt{3}=\boxed{-18\sqrt{5}+2\sqrt{3}}$

FOIL 12. $(2\sqrt{3}+\sqrt{5})(2\sqrt{3}-\sqrt{5})$

F: $(2\sqrt{3})(2\sqrt{3})=4\cdot3=12$

O: $(2\sqrt{3})(-\sqrt{5})=-2\sqrt{15}$

I: $(\sqrt{5})(2\sqrt{3})=2\sqrt{15}$

L: $(\sqrt{5})(-\sqrt{5})=-5$

$=12-2\sqrt{15}+2\sqrt{15}-5$

$=12-5=\boxed{7}$

13. $(3\sqrt{2}+1)(5-\sqrt{3})$ FOIL

F: $(3\sqrt{2})(5)=15\sqrt{2}$

O: $(3\sqrt{2})(-\sqrt{3})=-3\sqrt{6}$

I: $(1)(5)=5$

L: $(1)(-\sqrt{3})=-\sqrt{3}$

$\boxed{15\sqrt{2}-3\sqrt{6}+5-\sqrt{3}}$

Rationalize the denominator. ***Hint: To rationalize, multiply by a form of 1.***

14. $\frac{3}{\sqrt{7}}$

Multiply numerator and denominator by whatever radical is in the denominator.

$$\frac{3}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{3\sqrt{7}}{\sqrt{7}\sqrt{7}}=\frac{3\sqrt{7}}{\sqrt{49}}=\boxed{\frac{3\sqrt{7}}{7}}$$

15. $\frac{\sqrt{6}}{\sqrt{3}-4}$

Multiply the numerator and the denominator by the conjugate of the denominator

$\sqrt{3}-4 \xrightarrow{\text{conjugate}} \sqrt{3}+4$

$a+b \xrightarrow{\text{conjugate}} a-b$

$a-b \xrightarrow{\text{conjugate}} a+b$

$$\frac{\sqrt{6}}{\sqrt{3}-4}\cdot\frac{\sqrt{3}+4}{\sqrt{3}+4}=\frac{\sqrt{6}(\sqrt{3}+4)}{(\sqrt{3}-4)(\sqrt{3}+4)}$$

$$=\frac{\sqrt{6}\sqrt{3}+4\sqrt{6}}{\sqrt{3}\sqrt{3}+4\sqrt{3}-4\sqrt{3}-16}=\frac{\sqrt{18}+4\sqrt{6}}{3-16}=\frac{\sqrt{9}\sqrt{2}+4\sqrt{6}}{-13}=\boxed{\frac{3\sqrt{2}+4\sqrt{6}}{-13}}$$