Math 250 **Substitution Integration**

The following are nice to know (make your life easier) integrals:

$$\int \tan x dx = -\ln|\cos x| + c \qquad \int \csc x dx = -\ln|\csc x + \cot x| + c$$

$$\int \cot x dx = \ln|\sin x| + c \qquad \int \sec x dx = \ln|\sec x + \tan x| + c$$

If a is constant, $a \neq 0$:

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c \qquad \int \cos ax dx = \frac{1}{a} \sin ax + c \qquad \int \sin ax dx = -\frac{1}{a} \cos ax + c$$

Method 2. *Substitution.* A simple substitution reduces the integral to one of the first set of basic integrals. In fact, the second set came about by using substitution.

Example C. $\int \tan x dx = \int \frac{\sin x}{\cos x} dx$. Let $w = \cos x$ then $dw = -\sin x dx$ so that the integral becomes $\int -\frac{dw}{w} = -\ln|w| + c = -\ln|\cos x| + c$.

Example D. $\int x e^{x^2} dx$. Let $w = x^2$ then $dw = 2x dx$ so that the integral becomes $\int e^w \frac{dw}{2} = \frac{1}{2} e^w + c = \frac{1}{2} e^{x^2} + c$.

Hint I: Things to look for: if the integrand involves

$$e^{f(x)},\ \text{trig}\ (f(x)),\ \frac{1}{f(x)},\ (f(x))^n$$

Let $w = f(x)$. This is not an exclusive list!

Hint II: When substitution is complete, make sure no x's appear in the integrand.

Problems:

1. $\int \frac{\cos x}{1 + \sin x} dx$. Let $w = 1 + \sin x$.
2. $\int (3x^2 + x) \cos(2x^3 + x^2 + 4) dx$. Let $w = 2x^3 + x^2 + 4$.
3. $\int \frac{(\ln x)^3}{x} dx$. Let $w = \ln x$.
4. $\int \sec^2 x e^{\tan x} dx$
5. $\int x\sqrt{x+2} dx$

6. $\int \frac{x+1}{x^2+2x-5}dx$

7. Integrate.

   a) $\int r(1+r^2)^{3/2}dr$

   b) $\int \frac{x+1}{x^2+2x+4}dx$

   c) $\int \frac{x^2+3x-1}{x}dx$

   d) $\int (e^{4x}+\sin 3x)dx$

   e) $\int \frac{\sin x}{4+\cos x}dx$

   f) $\int \frac{\sec^2 x+\sec x\cot x}{\sec x+\tan x}dx$

8. Evaluate:

   a) $\int_0^1 \left(e^{x/2}-\cos\frac{\pi}{2}x\right)dx$

   b) $\int_0^1 x(1+x)^{1/2}dx$

9. Integrate.

   a) $\int (\sin 3x - e^{6x})dx$

   b) $\int xe^{x^2+1}dx$

   c) $\int \sqrt{x}(x-1)dx$

   d) $\int r(1+4r^2)^{1/2}dr$

   e) $\int \frac{(\ln x)^2}{x}dx$

   f) $\int \frac{x+1}{x^2+2x+4}dx$

10. Evaluate:

a) $\int_0^3 x(1+x)^{1/2}dx$

b) $\int_{-1}^1 x(1-x^2)^{1/2}dx$

11. Integrate.

a) $\int x\sin(2x^2+1)dx$

b) $\int(\sin 4x + \cos\frac{x}{2} - e^{-x})dx$

c) $\int \frac{x}{(1+x)^{1/2}}dx$

d) $\int \frac{(\ln x)^2}{x}dx$

e) $\int \frac{\sec x\tan x + \sec^2 x}{\sec x + \tan x}dx$

f) $\int \frac{dx}{1-\sin^2 x}dx$

12. Evaluate:

a) $\int_0^5 \frac{rdr}{(25-r^2)^{1/2}}$

b) $\int_0^1 \frac{xdx}{x^2+1}$

c) $\int_0^{\pi/4} e^{\tan x}\sec^2 x\,dx$