Math 250 **Integration by Parts**

Method 3. *Integration by parts.* This method undoes the product rule

$$\int udv = uv - \int vdu$$

Example A. $\int x \ln x dx$. Let $u = \ln x,\ dv = xdx$ then $du = \dfrac{dx}{x}$ and $v = \dfrac{x^2}{2}$.

$$\int x \ln x dx = (\ln x)\frac{x^2}{2} - \int \frac{x^2}{2}\frac{dx}{x} = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

Example B. $\int x \sin x dx$. Let $u = x,\ dv = \sin x dx$. Then $dv = dx$ and $v = -\cos x$.

$$\int x \sin x dx = x(-\cos x) - \int -\cos x dx = -x\cos x + \sin x + c$$

Hint I: When choosing u and dv make sure dv is something that can be integrated. Also, the whole integrand should be taken up with u and dv.

Hint II: Method should be used when integrand involves

(poly)trig	$e^{ax}(\sin bx)$ $e^{ax}\cos bx$	$\sec^{(2n+1)} x$
(poly) $\ln x$	poly(inverse trig fact)	$\csc^{(2n+1)} x$

This is not an exclusive list!

Problems:

1. $\int \ln x dx$. Let $u = \ln x,\ dv = dx$.

2. $\int (x^2 + 2x - 1)\cos 3x dx$. Let $u = x^2 + 2x - 1,\ dv = \cos 3x dx$.
   (Need to use integration by parts twice.)

3. $\int (x+3)e^{2x} dx$. Let $u = x + 3,\ dv = e^{2x} dx$.

4. $\int x^2 \ln x dx$

5. $\int \tan^{-1} x dx$

6. $\int e^x \sin x dx$

7. $\int (x+3)\sec^2 x dx$

8. $\int (x^2+1)e^{4x} dx$

9. $\int x^3 e^{x^2} dx$

10. $\int x \sin^{-1} x dx$

11. $\int \frac{\ln x}{x^3} dx$

12. $\int \sec^3 x dx$

13. $\int (x^2+1)\sin 2x dx$

14. $\int (x^2+x)\cos^2 x dx$

15. $\int (x^2+1)\sin x \cos x dx$

16. $\int (x+4)e^{1/2x} dx$

17. $\int \sin x \cos 2x dx$

18. $\int (x^2+1)\ln x dx$