Math 305 **Trigonometric Integrals**

Method 4. *Trigonometric integrals.* Integrands only involve trigonometric functions (not inverse trig functions!). Remember that certain trig functions "go together."

$$\sin\theta \text{ and } \cos\theta$$
$$\tan\theta \text{ and } \sec\theta$$
$$\cot\theta \text{ and } \cot\theta$$

If you have mixed trig functions convert everything to $\sin\theta$ and $\cos\theta$. Another helpful identity is $\sin^2\theta + \cos^2\theta = 1$. From here you can derive $\tan^2\theta + 1 = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$. You should also have the half angle formulas in your repertoire.

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

Most trigonometric integrations involve substitution. The only exceptions are when the integrand is $\sec^{2n+1}\theta$ or $\csc^{2n+1}\theta$ (see integration by parts) or $\sin^{2n}\theta\cos^{2m}\theta$ which involves the half angle formula (sometimes repeated several times).

Example A. $\int \sin\theta\cos^2\theta d\theta$. Let $w = \cos\theta; dw = -\sin\theta d\theta$

$$\int \sin\theta\cos^2\theta d\theta = \int -w^2 dw = -\frac{w^3}{3} + c = -\frac{\cos^3\theta}{3} + c$$

Example B. $\int \tan^2\theta d\theta = \int (\sec^2\theta - 1)d\theta = \tan\theta - \theta + c$

Example C. $\int \sin^2\theta\cos^3\theta d\theta$ Let $w = \sin\theta$ then $dw = \cos\theta d\theta$.

$$\int \sin^2\theta\cos^2\theta \underbrace{\sin\theta d\theta}_{dw} = \int w^2(1 - \sin^2\theta)dw =$$

$$\int w^2(1 - w^2)dw = \frac{w^3}{3} - \frac{w^5}{5} + c = \frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5} + c$$

Example D. $\int \sin^2\theta d\theta = \int \frac{1}{2}(1 - \cos 2\theta)d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + c$

Hint I: As you can see there's not a whole lot of guessing to do with substitution. For instance in Example I above if you had let $w = \cos\theta$ then $dw = -\sin\theta d\theta$ and you have an "extra" $\sin\theta$ in the integrand. Of course it would be silly to guess $w = \tan\theta$ or $\sec\theta$ or another trig function. Use common sense and a paper and pencil!

Problems:

1. $\int \tan\theta \sec^3\theta d\theta$
2. $\int \sin^2\theta \cos^2\theta d\theta$
3. $\int \tan^3 2\theta \sec^2 2\theta d\theta$
4. $\int \cot\theta \csc^3\theta d\theta$
5. $\int \tan^3\theta d\theta$
6. $\int \sin\theta \cos\theta d\theta$
7. $\int \cos^2\theta d\theta$
8. $\int \sin^3\theta d\theta$
9. $\int \sin^3\theta \cos^4\theta d\theta$
10. $\int \sin\theta \tan\theta d\theta$

Note that integrands involving different arguments were not covered. i.e., $\int \cos 2x \sin 3x dx$. These type of integrals can be done using integration by parts twice and bringing the integral to the other side.