Math 250 Review of Series Integral Test

Consider $\sum_{n=1}^{\infty} a_n$ where $a_n \geq 0$.

If $\int_1^{\infty} a_y dx < \infty \Rightarrow \sum_1^{\infty} a_n$ converges. If $\int_1^{\infty} a_x dx = \infty \Rightarrow \sum_1^{\infty} a_n$ diverges.

Example:

1. $\sum_1^{\infty} \frac{1}{n}$ $\int_1^{\infty} \frac{dx}{x} = \lim_{b\to\infty} [\ln b - \ln 1] = \infty \Rightarrow \sum_1^{\infty} \frac{1}{n}$ diverges

2. $\sum_1^{\infty} \frac{1}{n^2}$ $\int_1^{\infty} \frac{dx}{x} = \lim_{b\to\infty} \left[-\frac{1}{b} + \frac{1}{1} \right] = 1 < \infty \Rightarrow \sum_1^{\infty} \frac{1}{n^2}$ converges

Determine if the following series converge or diverge.

1. $\sum_1^{\infty} \frac{1}{n^2+1}$
2. $\sum_1^{\infty} \frac{1}{n^2-1}$
3. $\sum_1^{\infty} ne^{-n}$
4. $\sum_2^{\infty} \frac{1}{n \ln n}$
5. $\sum_1^{\infty} \frac{1}{n^2+5n+6}$
6. $\sum_2^{\infty} \frac{1}{n(\ln n)^3}$
7. $\sum_1^{\infty} \frac{\ln n}{n}$
8. $\sum_1^{\infty} \frac{\ln n}{2}$

Hint: $\int a_x dx$ needs to be something that you recognize as integrable.