Let $0 \le a_n \le b_n$. If $\sum_1^\infty b_n < \infty \Rightarrow \sum_1^\infty a_n$ converges.

If $\sum_1^\infty a_n = \infty \Rightarrow \sum_1^\infty b_n$ diverges.

E.g. 1) $\sum_1^\infty \frac{1}{n^2+1}$ Now $n^2+1 \ge n^2 \Rightarrow \frac{1}{n^2+1} \le \frac{1}{n^2}$ so that

$$\sum_1^\infty \frac{1}{n^2+1} \le \sum_1^\infty \frac{1}{n^2} < \infty \quad \left(\sum_1^\infty \frac{1}{n^2} \text{ converges, p-series } p = 2 > 1 \right)$$

2) $\sum_1^\infty \frac{1}{n^2-1}$ Now $n^2 - 1 \ge \frac{1}{2}n^2$ for $n > 1$ the

$\frac{1}{n^2-1} \le \frac{2}{n^2}$ so that

$$\sum_1^\infty \frac{1}{n^2-1} \le \sum \frac{2}{n^2} < \infty$$

Hint 1: You must again make an intelligent guess.

Hint 2: Every limit comparison test problem can be done with direct comparison test. Every direct comparison test problem cannot necessarily be done with limit comparison test.

3) $\sum_2^\infty \frac{1}{\ln n}$ Now $\ln n < n \Rightarrow \frac{1}{\ln n} > \frac{1}{n}$ and

$\infty = \sum_2^\infty \frac{1}{n} < \sum_2^\infty \frac{1}{\ln n}$ Since $\sum_2^\infty \frac{1}{n}$ diverges $\Rightarrow \sum_2^\infty \frac{1}{\ln n}$ diverges.

4) $\sum_1^\infty \frac{2+\sin n}{n^2}$ $-1 \le \sin n \le 1 \Rightarrow 1 \le 1 + \sin n \le 3 \Rightarrow \frac{1}{n^2} \le \frac{1+\sin n}{n^2} \le \frac{3}{n^2}$

Thus $\sum_1^\infty \frac{2+\sin n}{n^2} \le \sum \frac{3}{n^2} < \infty$

Determine whether the following converge or diverge.

1) $$\sum_{1}^{\infty} \frac{n}{n^3+4}$$

2) $$\sum_{1}^{\infty} \frac{3^n}{1+5^n}$$

3) $$\sum_{1}^{\infty} \frac{4+\cos n}{n^3}$$

4) $$\sum_{1}^{\infty} \frac{1}{n+2^n}$$

5) $$\sum_{1}^{\infty} \frac{5^n}{1+3^n}$$

6) $$\sum_{2}^{\infty} \frac{1}{(\ln n)^2}$$

7) $$\sum_{1}^{\infty} \sin\left(\frac{1}{n}\right)$$