Math 250 Series Review: Alternating Series Name______________________

Consider the series of the form $\sum_{1}^{\infty}(-1)^n a_n$ where $a_n \geq 0$.

A series converges if 1) Series alternates signs.

2) $\lim_{n\to\infty} a_n = 0$.

3) $a_{n+1} \leq a_n$.

E.g. 1) $\sum_{1}^{\infty}\frac{(-1)^n}{n}$ 1) Series alternates signs.

2) $\lim_{n\to\infty}\frac{1}{n} = 0$.

3) $n+1 \geq n \Rightarrow \frac{1}{n+1} \leq \frac{1}{n} \Rightarrow a_{n+1} \leq a_n$ So series converges.

E.g. 2) $\sum_{1}^{\infty}\frac{\cos n\pi}{n^2}$ 1) Series alternates signs. $(\cos\pi = (-1)^n)$

2) $\lim_{n\to\infty}\frac{1}{n^2} = 0$.

3) $(n+1)^2 \geq n^2 \Rightarrow \frac{1}{(n+1)^2} \leq \frac{1}{n^2} \Rightarrow a_{n+1} \leq a_n$. So series converges.

Hint: to determine if $a_{n+1} \leq a_n$ all you need to do is show $\frac{d}{dn}a_n < 0$. Then the series decreases (from Calculus 1).

Determine if the following series converges or diverges.

1) $\sum_{1}^{\infty}\left(-\frac{2}{3}\right)^{n}$

2) $\sum_{2}^{\infty}\frac{(-1)^{n}}{\ln n}$

3) $\sum_{2}^{\infty}\frac{(-1)^{n}}{n\ln n}$

4) $\sum_{1}^{\infty}\frac{(-1)^{2n}}{n}$

5) $\sum_{1}^{\infty}\frac{(-1)^{n}n}{\sqrt{n^{2}+1}}$

6) $\sum_{2}^{\infty}\frac{(-1)^{n}\ln n}{n}$