Series Review: Absolute Convergence, Conditional Convergence, And Divergence
Math 250

Consider $\sum_{1}^{\infty} a_n$

If $\sum_{1}^{\infty} |a_n|$ converges $\Rightarrow \sum_{1}^{\infty} a_n$ is absolutely convergent.

Or If $\sum_{1}^{\infty} |a_n|$ diverges and $\sum_{1}^{\infty} a_n$ converges $\Rightarrow \sum_{1}^{\infty} a_n$ is conditionally convergent.

Or $\sum_{1}^{\infty} a_n$ diverges.

E.g. 1) $\sum_{1}^{\infty} \frac{(-1)^n}{n}$. $\sum_{1}^{\infty} \frac{1}{n}$ diverges but $\sum_{1}^{\infty} \frac{(-1)^n}{n}$ converges A.S.T.

$\Rightarrow \sum_{1}^{\infty} \frac{(-1)^n}{n}$ is conditionally convergent

2) $\sum_{1}^{\infty} \frac{(-1)^n}{n^2}$ $\sum_{1}^{\infty} \frac{1}{n^2}$ converges $\left(\text{p series } p = 2 > 1\right)$

$\Rightarrow \sum_{1}^{\infty} \frac{(-1)^n}{n^2}$ is absolutely convergent.

3) $\sum_{1}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$ is divergent $\left(\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0\right)$

Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

1) $\sum_{1}^{\infty} \frac{(-1)^n}{1+n^{1/2}}$

2) $\sum_{1}^{\infty} \frac{(-1)^n}{1+n^2}$

3) $\sum_{1}^{\infty} \frac{(-2)^n}{1+3^n}$

4) $\sum_{1}^{\infty} \frac{\cos n\pi}{n\sqrt{\ln n}}$

5) $\sum_{1}^{\infty} \frac{(-1)^{2n}}{n^{1/3}}$

6) $\sum_{1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n}$