

Consider  $\sum_1^{\infty} a_n$ . Let  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$  if

$r < 1 \Rightarrow$  series is absolutely convergent

$r > 1 \Rightarrow$  series diverges

$r = 1 \Rightarrow$  wrong test

E.g., 1)  $\sum_1^{\infty} \frac{1}{n!} \lim_{n \rightarrow \infty} \left| \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} \right| = \lim_{n \rightarrow \infty} \frac{n!}{(n+1)!} = \frac{1}{n+1} = 0 < 1$

$\Rightarrow \sum_1^{\infty} \frac{1}{n}$  converges.

2)  $\sum_1^{\infty} \frac{n}{2^n} \lim_{n \rightarrow \infty} \left| \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} \right|$   
 $= \lim_{n \rightarrow \infty} \left| \frac{n+1}{n} \right| \frac{1}{2} = \frac{1}{2} < 1 \Rightarrow \sum_1^{\infty} \frac{n}{2^n}$  converges.

Determine if the following series are absolutely convergent, conditionally convergent, or divergent:

$$\sum_1^{\infty} \frac{(-2)^n}{n^2 + 1}$$

$$\sum_1^{\infty} n e^{-n}$$

$$\sum_1^{\infty} \frac{n!}{n^n}$$

$$\sum_0^{\infty} \frac{n!n!}{(2n)!}$$

$$\sum_1^{\infty} \frac{3^n}{n!}$$

$$\sum_1^{\infty} \frac{n!}{2^n}$$