

This method is the most straightforward of all the methods. Basically, the ODE can be written as

$$y' = f(t)g(y).$$

As you can see the variables “separate” into the product of two different functions. From here you divide by  $g(y)$ , multiply by  $dt$ , and integrate both sides.

Example A.  $y' = t^2 \cos y$       $y(1) = 0$

1. Notice  $\frac{dy}{dt} = t^2 \cos y$  so that  $\frac{dy}{\cos y} = t^2 dt$ .
2. Integrating  $\int \sec y dy = \int t^2 dt$  so that  $\ln |\sec y + \tan y| = \frac{t^3}{3} + c$
3. Plugging in IC:  $\ln 1 = \frac{1^3}{3} + c$
4. Thus  $\ln |\sec y + \tan y| = \frac{t^3}{3} - \frac{1}{3}$ .

Example B.  $y' = y^2 \sin t + \sin t + y^2 e^t + e^t$       $y(0) = 1$

1. Rewrite the ODE as  $\frac{dy}{dt} = (y^2 + 1)(\sin t + e^t)$ .
2. Divide by  $y^2 + 1$ , multiply by  $dt$  to get  $\frac{dy}{y^2 + 1} = (\sin t + e^t)dt$ .
3. Integrating both sides we have  $\int \frac{dy}{y^2 + 1} = \tan^{-1} y = \int (\sin t + e^t)dt = -\cos t + e^t + c$ .
4. Plugging in IC:  $\tan^{-1} 1 = -\cos 0 + e^0 + c$ .
5. Thus  $\tan^{-1} y = -\cos t + e^t + \frac{\pi}{4}$ .

Hint I: As you can see, probably the hardest part of this problem is factoring the right-hand side!

**Problems:**

1.  $y' = t^3 e^y + t e^y + e^y \quad y(1) = \ln 2$

2.  $y' = ty + y \tan t + t + \tan t \quad y(1) = 0$

3.  $y' = t^2 - t \cos y - 1 + t + \cos y - t^2 \cos y \quad y(2) = \frac{\pi}{4}$

4.  $t' = e^{t+y} \quad y(\ln 2) = \ln 3$

5.  $y' = \frac{e^t + 1}{y^2 + 2} \quad y(0) = -3$

**Solutions:**

1.  $-e^{-y} = \frac{t^4}{4} = \frac{t^2}{2} + t - \frac{9}{4}$

2.  $\ln(y + 1) = -\ln \sec t + \frac{t^2}{2} + \ln \sec t - \frac{1}{2}$

3.  $-\cot y - \csc y = \frac{t^3}{3} + \frac{t^2}{2} - t - \sqrt{2} - \frac{4}{3}$

4.  $e^y = -e^{-t} + \frac{7}{2}$

5.  $\frac{y^3}{3} + 2y = e^t + t + 2$