

A first order homogeneous equation can be written in the form

$$y' = f\left(\frac{y}{t}\right)$$

Basically if y appears then it needs to be divided by t .

Steps:

1. Make sure the equation is homogeneous.
2. Let $v(t) = \frac{y(t)}{t}$.
3. Rewrite $y(t) = tv(t)$, differentiate with respect to t : $\frac{dy}{dt} = v + t\frac{dv}{dt}$.
4. Substitute the above in for $\frac{dy}{dt}$, rewrite all $\frac{y}{t}$ as v .
5. Equation is now separable.

Example A. $ty' = y + t \cos \frac{y}{t}$

1. Notice the equation can be rewritten as $y' = \frac{y}{t} + \cos \frac{y}{t}$.
2. $v(t) = \frac{y(t)}{t}$
3. $y(t) = tv(t)$ so that $\frac{dy}{dt} = v + t\frac{dv}{dt}$
4. $v + t\frac{dv}{dt} = v + \cos v$
5. $t\frac{dv}{dt} = \cos v$ Now it's separable. Can you finish the problem?

Example B. $y' = \frac{t-y}{t+y}$

1. Notice equation can be written as $y' = \frac{1 - \frac{y}{t}}{1 + \frac{y}{t}}$.

2. $v(t) = \frac{y(t)}{t}$

3. $y(t) = tv(t)$ so that $\frac{dy}{dt} = v + t\frac{dv}{dt}$

4. $v + t\frac{dv}{dt} = \frac{1-v}{1+v}$

5. $t\frac{dv}{dt} = \frac{1-2v-v^2}{1+v}$

Problems:

1. $ty' - y = te^{y/t} \quad y(1) = 0$

2. $y' = \frac{t+y}{t-2y} \quad y(1) = 3$

3. $y' = \frac{4t+3y}{3t+y} \quad y(0) = 1$

4. $ty' - y = \frac{t^2+ty}{t-y} \quad y(1) = 2$

5. $y' = \frac{yt^2 - 8t^3}{t^3 + y^3} \quad y(1) = 3$

Solutions:

1. $e^{-y/t} = -\ln t + 1$

2. $\frac{1}{\sqrt{2}} \tan^{-1} \frac{\sqrt{2}y}{t} = -\ln t + \frac{1}{\sqrt{2}} \tan^{-1} 3\sqrt{2} - \ln 10$

3. $(2t+y) = -(2t-y)^5$

4. $-\frac{y}{t} + 2 \ln \left(1 + \frac{y}{t}\right) = \ln t - 2 + \ln 3$

5. Work in progress.