Math 305 **Reduction of Order**

To use reduction of order you must have linear, homogeneous ODEs, (be of the form $y''+p(t)y'+q(t)y=0$) and you must know one solution (y).

(Be a good guesser!!) You look for a second solution of the form $y = vy_1$. Differentiate and plug with the ODE. There will be no v's appearring explicitly in equation. Solve using the methods of higher order equations.

Example A. $t^2y''+ty'-y=0$ y_1t is a solution. (Note $y_1=t, y_1'=1, y_1''=0$ and $t^2-0+t-1-t=0$.)

1. Let $y = tv$.

2. Differentiate: $y' = v + tv'$ and $y'' = 2v' + tv''$.

3. Plug into ODE: $t^2(2v'+tv'')+t(v+tv')-tv=0$

4. Simplify: $v''(t^3)+v'(3t^2)=0$ (Notice no v term.)

5. Let $v' = w(t)$. Then $\dfrac{dv'}{dt} = v'' = \dfrac{dw}{dt}$.

6. Plug into ODE: $t^3\dfrac{dw}{dt}+3t^2w=0$

7. Solve: This is linear and separable so $\dfrac{d}{dt}(t^3w)=0$ and $t^3w=c_1$.

8. Plug $w = v'$: $\dfrac{dv}{dt} = \dfrac{c_1}{t^3}$

9. Solve: $v = -\dfrac{c_1}{2t^2}+c_2$

10. Plug but $y = tv = t\left(-\dfrac{c_1}{2t^2}+c_2\right) = -\dfrac{c_1}{2}\dfrac{1}{t}+c_2t$.

11. Two different solutions are: t and $\dfrac{1}{t}$.

12. Homogeneous Equation: $y_n = c_1t + c_2\cdot\dfrac{1}{t}$.

Example B. $t^2y'' + 3ty' - 3y = 0$. (Note $y_1 = t^{-3}$ is a solution.)

1. Let $y = t^{-3}v$.

2. Differentiate: $y' = -3t^{-4}v + t^{-3}v^1$ and $y'' = 12t^{-5}v - 6t^{-4}v' + t^{-3}v''$.

3. Plug into ODE: $t^2(12t^{-5}v - 6t^{-4}v' + t^{-3}v'') + 3t(-3t^{-4}v + t^{-3}v') - 3t^{-3}v = 0$

4. Simplify: $v''(t^{-2}) + v'(-6t^{-2} + 3t^{-2}) = 0$ or $v''t^{-2} + v'(-3t^{-2}) = 0$

5. Let $v' = w(t)$. Then $v'' = \dfrac{dw}{dt}$.

6. Plug into ODE: $t^{-3}\dfrac{dw}{dt} - 3t^{-4}w = 0$

7. Solve: $\dfrac{d}{dt}(t^{-3}w) = 0$ and $t^{-3}w = c_1$.

8. Plug $w = v'$: $\dfrac{dv}{dt} = c_1t^3$

9. Solve: $v = c_1\dfrac{t_4}{4} + c_2$

10. Plug $y = t^{-3}\left(\dfrac{c_1}{4}t^4 + c_2\right) = \dfrac{c_1}{4}t + c_2t^{-3}$.

11. Two different solutions are: t^{-3} and t

12. Homogeneous Equation: $y_n = c_1t^{-3} + c_2t$

Problems:

1. $ty'' - (2t+1)y' + (t+1)y = 0 \quad y_1 = e^t$

2. $y'' - (2\cot t)y' + (1 + 2\cot^2 t)y = 0 \qquad y_1 = \sin t$

3. $(t+1)^2y'' - 4(t+1)y' + 6y = 0 \quad y_1 = (t+1)^2$

4. $y'' + 4ty' + (2 + 4t^2)y = 0 \quad y_1 = e^{-t^2}$

5. $y'' - \left(2t + \dfrac{1}{t}\right)y' = 0 \qquad y_1 = e^{t^2}$