

The solutions to problems 1 through 5 take the form

$$u(x,t) = \sum_1^{\infty} c_n e^{-n^2 \pi^2 \alpha^2 t / l^2} \sin \frac{n \pi x}{l},$$

where  $u_t = \alpha^2 u_{xx}$  and  $u(0,t) = u(l,t) = 0$ . solve for the  $c_n$ 's.

1)  $u_t = 4u_{xx}$

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = \sum_1^{20} \frac{1}{n^2} \sin n \pi x$$

4)  $u_t = \frac{1}{7} u_{xx}$

$$u(0,t) = u(7,t) = 0$$

$$u(x,0) = \sum_1^{500} \frac{1}{n!} \sin n \pi x$$

2)  $4u_t = u_{xx}$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = \sum_1^{20} \frac{1}{n^2} \sin n \pi x$$

5)  $\frac{1}{7} u_t = u_{xx}$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = \sum_1^{\infty} \frac{1}{n^n} \sin 2n \pi x$$

3)  $u_t = 3u_{xx}$

$$u(0,t) = u(6,t) = 0$$

$$u(x,0) = \sum_1^{100} (-1)^n \sin \frac{n \pi x}{6}$$

In each of the problems 6 through 9 write the given function as a Fourier sine series.

6)  $f(x) = x(x-1), 0 \leq x < 1$

8)  $f(x) = \sin 3\pi x, 0 \leq x < \frac{2}{3}$

7)  $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x < 2 \end{cases}$

9)  $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x < 3 \\ x-4, & 3 \leq x < 4 \end{cases}$

Now solve problems 10 – 13:

10)  $u_t = 3u_{xx}$

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = x(x-1)$$

12)  $7u_t = u_{xx}$

$$u(0,t) = u(2/3,t) = 0$$

$$u(x,0) = \sin 3\pi x$$

11)  $u_t = \frac{1}{7} u_{xx}$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x < 2 \end{cases}$$

13)  $\frac{1}{6} u_t = u_{xx}$

$$u(t,0) = u(4,t) = 0$$

$$u(x,0) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x < 3 \\ x-4, & 3 \leq x < 4 \end{cases}$$

Solve:

14)  $u_t = u_{xx}$

$$u_x(0,t) = u_x(2,t) = 0$$

$$u(x,0) = 1$$

15)  $u_t = u_{xx}$

$$u_x(0,t) = u(1,t) = 0$$

$$u(x,0) = 1$$