SECTION 2.2 – THE COMPLEX NUMBERS

It is time to examine numbers that are not real.

- The number i is defined such that $i = \sqrt{-1}$ and $i^2 = -1$. Numbers containing i will be called **complex numbers.**
- Any complex number can be written in the form $a + bi$ where a and b are real numbers.
- We can only add and subtract real numbers with real numbers and imaginary numbers with imaginary numbers.
- When performing operations i "acts like" a variable.

Ex. 1: Simplify. Write answers in $a + bi$ form.

a) $(2+3i) - (5 - 2i)$

$= 2 + 3i - 5 + 2i$

$= -3 + 5i$

b) $(3 - 2i)^2$

$(3-2i)(3-2i)$

$= 9 - 6i - 6i + 4i^2$

$= 9 - 12i + 4(-1)$

$= 5 - 12i$

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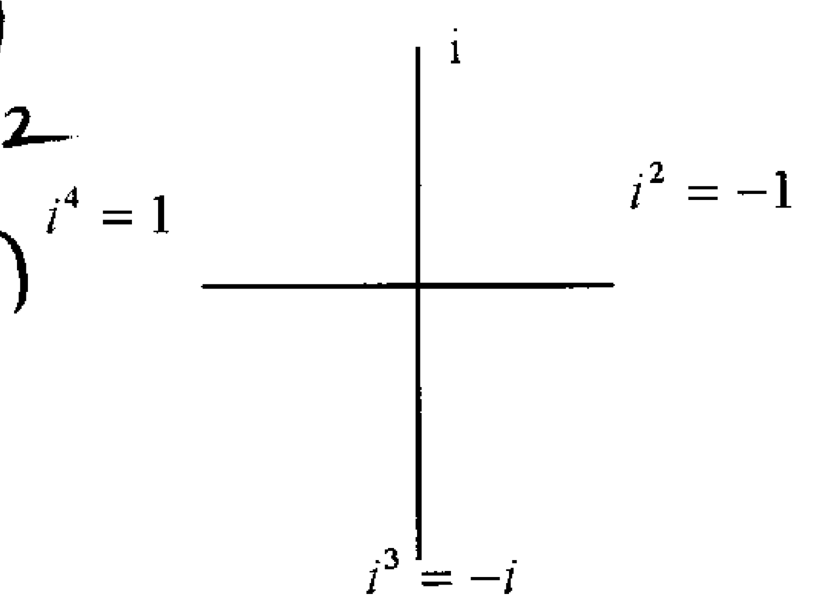


c) i^{10}

$= (i^2)^5$

$= (-1)^5 = -1$

d) $(5i)^3$

$= 5^3 i^3$

$= 125 \cdot \underbrace{i^2}_{-1} \cdot i = -125i$

Ex. 2: Simplify.

a) $-\sqrt{4}$

$= -2$

b) $\sqrt{-4}$

$= \sqrt{i^2 4}$

$= \mp 2i$

c) $\sqrt{-10} = \sqrt{i^2 \cdot 10}$

$= \sqrt{10}\, i$

d) $\dfrac{4 \pm \sqrt{-20}}{2}$

$= \dfrac{4 \mp \sqrt{i^2(4)(5)}}{2}$

$= \dfrac{4 \mp 2\sqrt{5}\, i}{2} = \dfrac{\cancel{2}(2 \mp \sqrt{5}\, i)}{\cancel{2}}$

$= 2 \mp \sqrt{5}\, i$

Division of complex numbers should remind you of the process called "rationalizing". Notice what happens when any $a+bi$ is multiplied by $a-bi$.

a+bi	*a–bi*	*(a+bi)(a–bi)*
$2+i$	$2-i$	$4-i^2=4+1=5$
$-1+2i$	$-1-2i$	$1-4i^2=1-4(-1)=5$
$3-i$	$3+i$	$9-i^2=9+1=10$

Conclusion: Multiplying any complex number by its conjugate will always yield a real number.

Ex. 3: Simplify and write in $a+bi$ form. $\dfrac{(5+2i)(3+i)}{(3-i)(3+i)} = \dfrac{15+5i+6i+2i^2}{9-i^2}$

$i^2 \to -1$

$$= \frac{15+11i-2}{9-(-1)} = \frac{13+11i}{10} = \frac{13}{10} + \frac{11}{10}i$$

Warning: Complex numbers will be used throughout the remainder of the course. However, our discussion of functions, particularly in regards to domain, is restricted to *real* numbers.

Suppose you are given $f(x)=\sqrt{x-3}$; $f(2)=\sqrt{2-3}=\sqrt{-1}$ (which is not real). That means 2 is not in the domain of $f(x)$. In fact, the domain of f is (still) $\{x \mid x \geq 3\}$.

Additional Notes/Written Homework

Recommended homework:
Section 2.2: #9, 21, 41, 49, 51, 63, 75, 79, 94

Problem Solving:

Ever since students started doing math on the walls of caves, word problems have been their Achilles heel. Applications are a major part of this course, so let's work together and improve.

Here is a plan/strategy to follow when setting up a word problem.

1) Read the problem *carefully!*

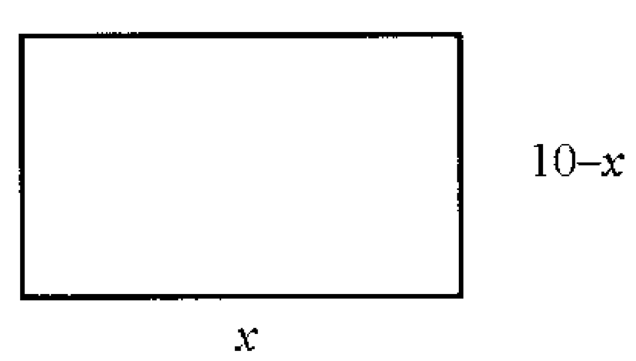
2) Read the problem AGAIN. This time, decide:
   - what is/are the unknown(s)?
   - what are the given quantities?
   - what are the given conditions?

3) For some problems (mixture, distance-rate-time) a chart will be helpful. For others, such as area, perimeter, volume, sides of a triangle, a diagram or sketch will be useful.

4) Find the relationship between what is known and unknown. This could be a formula such as $d = rt$, $V = lwh$, $a^2 + b^2 = c^2$... or even a common physical relationship (current will speed up a boat when moving downstream and slow it down when moving upstream).

5) Express the relationship as an equation or function.

   For example, if given the dimensions of a rectangle,



   a) The area can be expressed as a function of x as $A(x) = x(10 - x)$

   or

   b) If the area is given to be 70 square units, you can find the dimensions by setting up and solving:
   $x(10 - x) = 70$

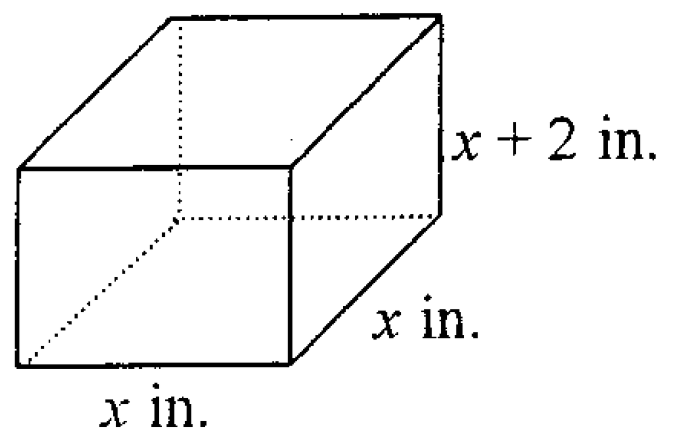
6) Answer the question asked.

   For example, if asked to find the dimensions that yield maximum area, be sure to indicate both the length and the width.

7) Practice, practice, practice. A plan helps, but so does time and effort!

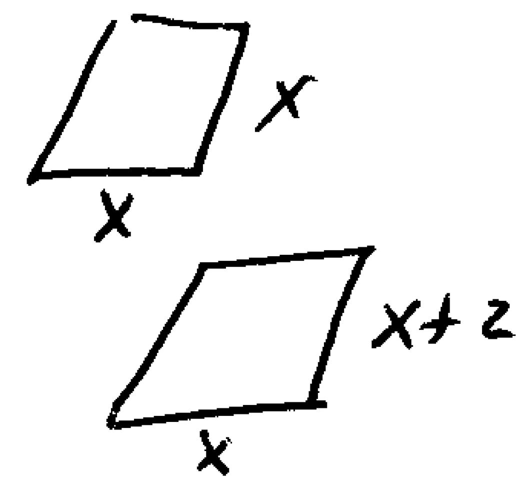
Miscellaneous Word Problems

Ex. 1: Find the surface area and volume of the $\boxed{\text{open box}}$ below.



Base ⇒

4 "sides ⇒



Note: Surface area is the sum of the areas of all surfaces. An open box has 5 surfaces.

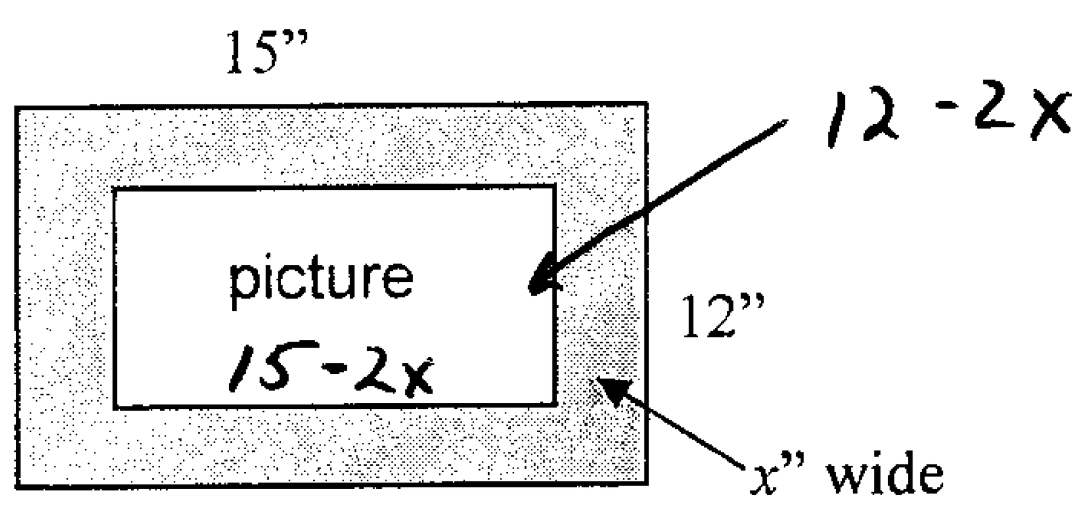
$SA(x) = \quad x^2 + 4x(x+2) \quad \text{or} \quad 5x^2 + 8x$

$V(x) = \quad lwh = x(x)(x+2)$

$= x^2(x+2)$

$\text{or} \quad x^3 + 2x^2$

Ex. 2: A picture is surrounded by a frame of uniform width. In terms of x, find the area of the frame.



$A(x) =$

Length and width of picture:

length $= 15 - 2x$

width $= 12 - 2x$

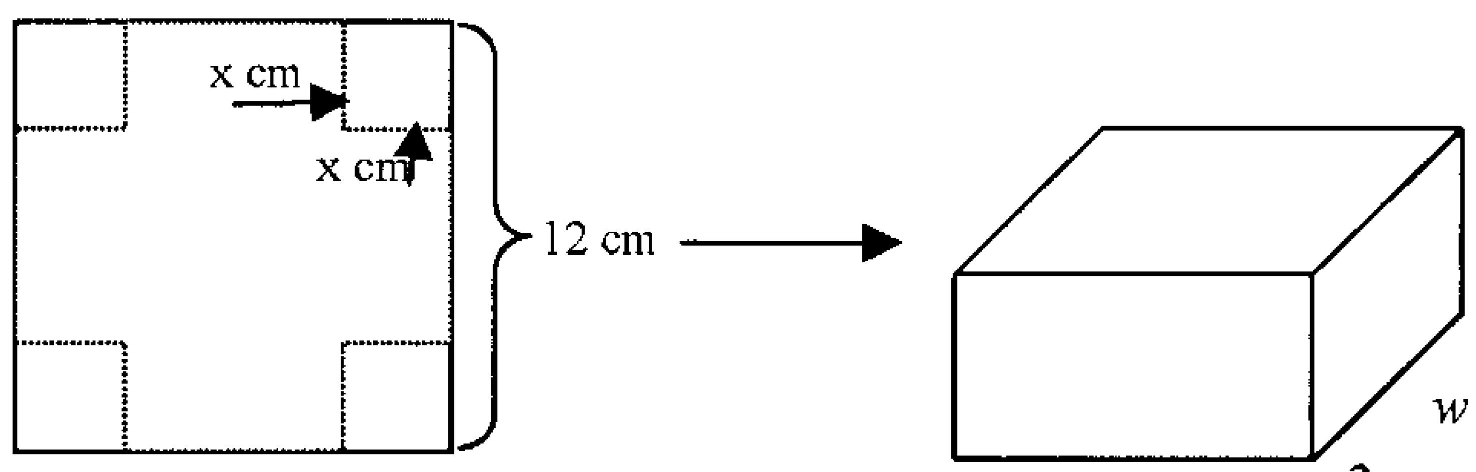
$A(x) = A_{\text{pic and frame}} - A_{\text{pic}}$

$= 15(12) - (15-2x)(12-2x)$

$= 180 - (180 - 54x + 4x^2) = 54x - 4x^2$

$\text{or} \quad 2x(27-2x)$

Ex. 3: A square piece of sheet metal has square corners of side x removed allowing it to be folded into an open box. Find the volume of the box as a function of x.



$h = x$

$w = 12 - 2x$

$\ell = 12 - 2x$

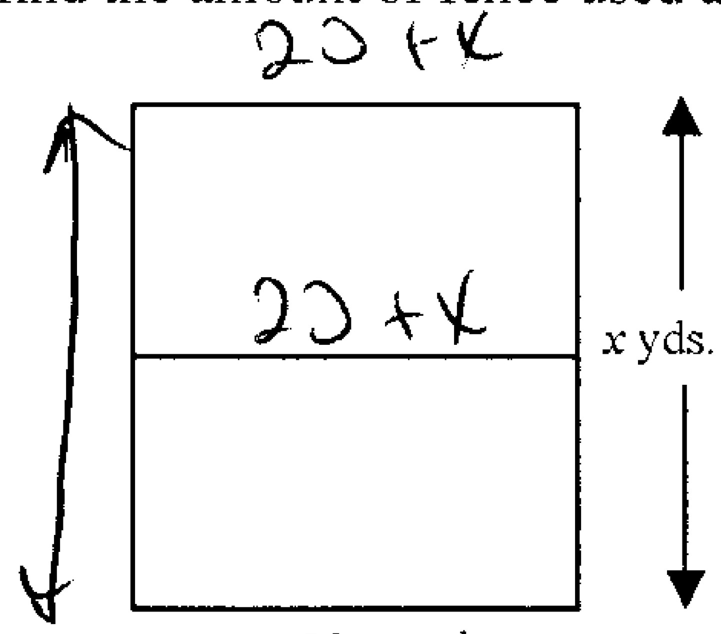
$V = \ell \cdot w \cdot h$

$V(x) = (12 - 2x)(12 - 2x)\, x$

$V = (144 - 24x - 24x - 4x^2)\, x$

$V = 144x - 48x^2 - 4x^3$

Ex. 4: A corral is fenced in and divided as indicated in the picture below. In terms of x, find the amount of fence used and the area of the enclosed region.



20+x yds

Amount of fence

$(20+x)+(20+x)+(20+x)+x+x$

$= 60 + 5x$ yrds

Area $= \ell \times w$

$= (20+x)\, x$

$= 20x + x^2$