SECTION 2.2-2.3 – THE ALGEBRA OF FUNCTIONS; COMPOSITION

Operations with functions:

$(f+g)(x) = f(x)+g(x)$
$(f-g)(x) = f(x)-g(x)$
$(fg)(x) = f(x)g(x)$
$(f/g)(x) = f(x)/g(x)$

Composition of functions:

$(f \circ g)(x) = f(g(x))$ Read as "f of g of x"
$(g \circ f)(x) = g(f(x))$

Note: (fg) is for multiplication whereas $(f \circ g)$ is for composition!!

Ex. 1: If $f(x) = 2x-6$ and $g(x) = x^2-5$, find:

a) $(f+g)(3)$

$= f(3)+g(3)$
$= 0 + 4$
$= 4$

b) $(f-g)(x)$

$= f(x) - g(x)$
$= 2x-6 - (x^2-5)$
$= -x^2+2x-1$

c) $\left(\frac{f}{g}\right)(-1)$

$= \dfrac{f(-1)}{g(-1)}$

$= \dfrac{-8}{-4} = 2$

$g(-1) = (-1)^2-5$
$= -4$

d) $(fg)(x)$

$= (2x-6)(x^2-5)$
$= 2x^3-6x^2-10x+30$

e) Is 3 in the domain of g ?

Is 3 in the domain of f?

Is 3 in the domain of $\frac{g}{f}$?

Ex. 2: Suppose $f(x)=x^2+2$ and $g(x)=\sqrt{x-5}$, find:

a) $(f \circ g)(x)$ "put g into f"

$= \left(\sqrt{x-5}\right)^2+2$

$= x-5+2 = x-3$

b) $g(f(3))$ Find $f(3)$

$f(3) = 3^2+2 = 11$

now find $g(11)$

$g(11) = \sqrt{6}$

c) Is 1 in the domain of f? Yes $f(1)=3$

Is 1 in the domain of $f \circ g$? $g \circ f$

NO

$(f \circ g)(1)$

Ex. 3: Find and simplify $\dfrac{f(x)-f(2)}{x-2}$ given that $f(x)=x^2-3x$.

Set up $\Rightarrow \dfrac{(x^2-3x)-(-2)}{x-2}$

Simplify $\Rightarrow \dfrac{x^2-3x+2}{x-2}$

Factor/Cancel $\Rightarrow \dfrac{(x-2)(x-1)}{x-2}$

$= x-1$

Note: $f(2)=2^2-3(2)$

$=-2$

Difference Quotients:

The expression used in Example 3 is called a **difference quotient**. In calculus, it is used to calculate slope. There are several forms of this quotient, but for our study we will only use $\dfrac{f(x+h)-f(x)}{h}$.

Before we start, here are two common errors. Try not to make them.

1) $f(x+h) \neq f(x)+h$! $(x+h)$ is a quantity and like any input, must be substituted for each x in the function.

2) Oftentimes $f(x)$ has many terms. When subtracting $f(x)$, be sure to negate each term!

Ex. 4: Set up and simplify $\dfrac{f(x+h)-f(x)}{h}$ for each.

b) $f(x) = 5x^2 + 4x$

$$f(x+h) = 5(x+h)^2 + 4(x+h)$$

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{5(x+h)^2 + 4(x+h) - (5x^2+4x)}{h}$$

$$= \frac{5(x^2+2xh+h^2) + 4x + 4h - 5x^2 - 4x}{h}$$

$$= \frac{10xh + 5h^2 + 4h}{h}$$

$$= \frac{\cancel{h}(10x + 5h + 4)}{\cancel{h}}$$

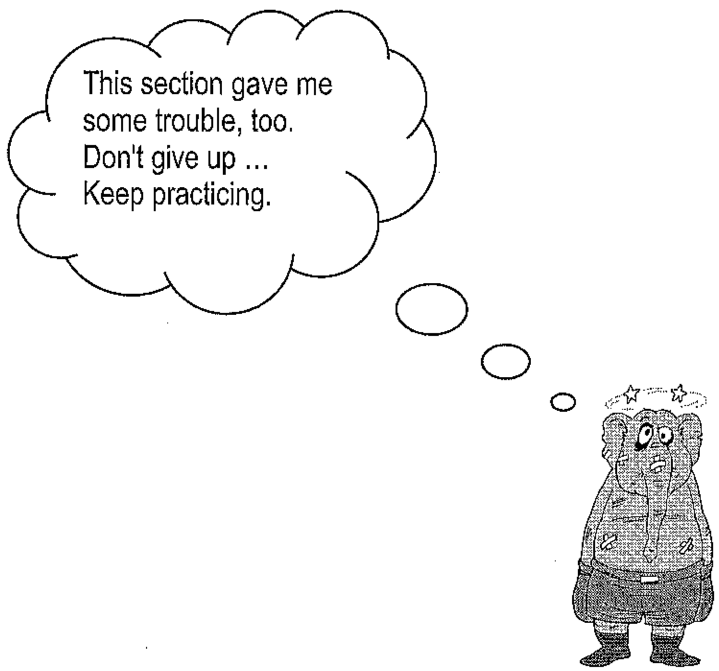
b) $f(x) = x^3$

$$f(x+h) = (x+h)^3$$

$$= x^3 + 3x^2h + 3xh^2 + h^3$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$



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e) Is 3 in the domain of g ?

Is 3 in the domain of f?

Is 3 in the domain of $\dfrac{g}{f}$?

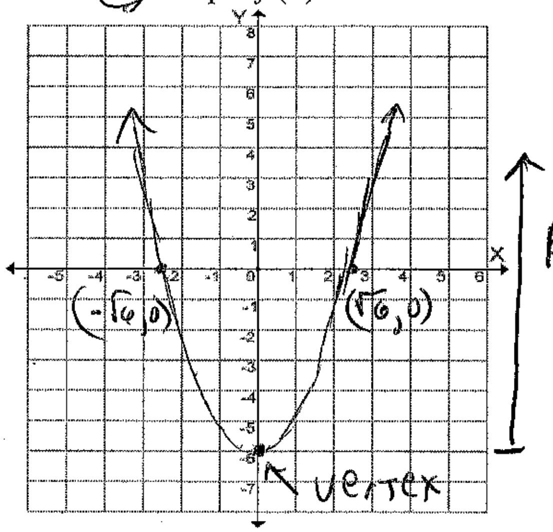
$f(0) = -6$

$x^2 - 6 = 0$

at $\pm\sqrt{6}$

Synthesis: To read intervals you first must graph.

Ex. 2: Graph $f(x) = x^2 - 6$.



State the following:

f is increasing on : $(0, \infty)$

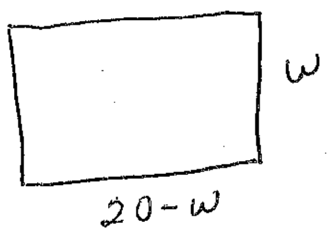
RANGE f is decreasing on : $(-\infty, 0)$

Relative ~~max~~/min is -6 when $x =$ 0

The domain is: $(-\infty, \infty)$

The range is: $[-6, \infty)$

Ex. 3: Dan uses 40 ft of fence to enclose his rectangular garden. If the garden is w feet wide, express its area as a function of the width.



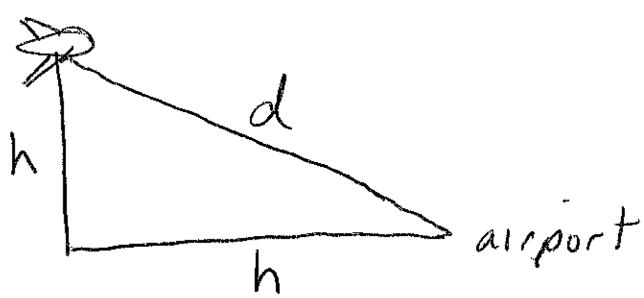
40 ft to enclose 4 sides

20 ft to enclose length and width

$A = lw$

$= w(20-w)$

Ex. 4: An airplane is flying at an altitude of 3700 ft. The slanted distance directly to the airport is d feet. Express the horizontal distance h as a function of d.



$h^2 + 3700^2 = d^2$

$h^2 = d^2 - 3700^2$

$h = \sqrt{d^2 - 3700^2}$

Sec. 1.6 – Solving Linear Inequalities

I. Linear Inequalities

The steps used to solve linear inequalities are nearly identical to those used to solve linear equations. There is one exception. When multiplying or dividing through by a negative number, be sure to reverse the sign of the inequality.

To demonstrate why inequalities are reversed:

Start with $-2 > -4$.

Multiply through by –3: $-2(-3) > -4(-3)$

Result: $6 < 12$

The same is true when dividing by a negative.

When multiplying or dividing an inequality by a negative, reverse the order.

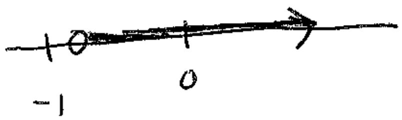
Ex. 1: Solve and graph the solution set. Give your answer in both set notation and interval notation.

1) $3 - x < 4x + 7$

$-x - 4x < 7 - 3$

$\frac{-5x}{-5} < \frac{4}{-5}$

$x > -\frac{4}{5}$



$(-\frac{4}{5}, \infty)$

2) $-3 < \frac{1-2x}{2} \le 3$

When solving a compound inequality, get x "alone" in the middle.

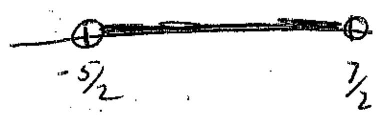
$2(-3) < 1 - 2x \le 2(3)$

$-6 < 1 - 2x \le 6$

$-1 \qquad\qquad -1$

$\frac{-7}{-2} < \frac{-2x}{-2} \le \frac{5}{-2}$

$\frac{7}{2} > x > -\frac{5}{2}$ or $-\frac{5}{2} < x < \frac{7}{2}$



$(-\frac{5}{2}, \frac{7}{2})$

SECTION 1.5 – Linear Equations

I. Linear Equations

Remember, to solve for a variable x means to get it in "$x =$" form. The variable being solved for cannot appear in the solution.

Ex. 1: Solve for the indicated variable. Start by clearing fractions.

a) $\frac{x-1}{4} - \frac{2x+3}{5} = 4$

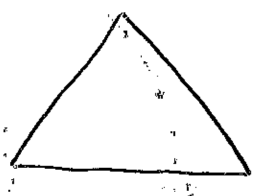
$\overset{5}{\cancel{20}}\left(\frac{x-1}{\cancel{4}}\right) - \overset{4}{\cancel{20}}\left(\frac{2x+3}{\cancel{5}}\right) = 4(20)$	LCD = 20
$5(x-1) - 4(2x+3) = 80$	Cancel
$5x - 5 - 8x - 12 = 80$	Remove parentheses
$-3x = 97$	
$x = -\frac{97}{3}$	

b) $A = \frac{1}{2}h(b_1 + b_2)$ for b_1

$$2(A) = 2\left(\tfrac{1}{2}h(b_1 + b_2)\right)$$
$$2A = hb_1 + hb_2$$
$$2A - hb_2 = hb_1$$
$$\frac{2A - hb_2}{h} = b_1$$

Ex. 2: In triangle ABC, angle B is twice as large as angle A. Angle C measures 20 degrees more than angle A. Find the measures of the angles.

Need to know: the sum of the angles in a triangle is 180 degrees!



x = measure of $\angle A$
$2x$ = measure of $\angle B$
$x+20$ = measure of $\angle C$

$$x + 2x + x + 20 = 180$$
$$4x = 160$$
$$x = 40$$

$\angle A = 40$
$\angle B = 2(40) = 80$
$\angle C = 40 + 20 = 60$

Check
$40 + 80 + 60 = 180^\circ$