R.6 Rational Expressions

Remember, a rational expression is the quotient of two polynomials: (from *R*.5)

Examples: $\frac{3}{5}$ $\frac{4}{x+1}$ $\frac{3x^2+4}{3x-4}$

Rules for canceling.

- Only factors can be cancelled
- Terms may NOT be cancelled.

Ex. 9: Express in simplest form:

a) $\frac{x(x+3)}{3x} = \boxed{\frac{x+3}{3}}$ b) $\frac{x\cancel{(x-3)}^{-1}}{\cancel{3-x}} = -x$ c) $\frac{x+3}{x}$ $\boxed{\text{in simplest form}}$

d) $\frac{x-3x^2}{x^2} = \boxed{\frac{x(x-3)}{x^2} = \frac{x-3}{x}}$ e) $\frac{x^2-4}{x^2-5x+6} = \frac{(x-2)(x+2)}{(x-2)(x-3)}$

$= \frac{x+2}{x-3}$

The **domain** of a rational expression includes all real numbers EXCEPT those values that make the denominator = 0.

Ex. 10: What is the domain of: $\frac{x+1}{2-x}$? D: $x \neq -2$

or $\{x \mid x \neq -2\}$ or $(-\infty,-2) \cup (-2,\infty)$

why $f(2) = \frac{3}{0}$ "undefined"

To **multiply or divide** rational expressions:

1) Factor all numerators and denominators.
2) Convert division into multiplication: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
3) Before multiplying, look to cancel factors.
4) Multiply. Factored form is preferred.

Multiplication does not require an LCD.

To **add or subtract** rational expressions:

1) Factor any denominators (not numerators).
2) Find the least common denominator (LCD).
3) Convert all expressions to have the same denominator.
4) Add and/or subtract numerators and place over common denominator. Leave denominators in factored form.
5) Simplify as shown in earlier examples.

Ex. 12: Perform the indicated operations and simplify:

a) $\dfrac{2}{x^2-9} - \dfrac{5}{x+3} \cdot \dfrac{(x-3)}{(x-3)}$ LCD $= (x-3)(x+3)$

$(x-3)(x+3)$

$= \dfrac{2 - 5x + 15}{(x-3)(x+3)}$

$= \dfrac{17-5x}{(x-3)(x+3)}$

b) $\dfrac{2(x+h)-(x+h)^2-(2x-x^2)}{h}$

$= \dfrac{2x+2h-(x^2+2xh+h^2)-2x+x^2}{h}$

$= \dfrac{\cancel{2x}+2h-\cancel{x^2}-2xh-h^2-\cancel{2x}+\cancel{x^2}}{h}$

$= \dfrac{h(2-2x-h)}{h} = 2-2x-h$

R.7 Radicals

Properties of Radicals (*R*.6)

Property	Example
1) If n is even, $\sqrt[n]{a^n} = \lvert a \rvert$	$\sqrt{x^2} = \lvert x \rvert$
2) If n is odd, $\sqrt[n]{a^n} = a$	$\sqrt[3]{a^3} = a$
3) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{7} \cdot \sqrt{2} = \sqrt{14}$
4) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$	$\dfrac{\sqrt{21}}{\sqrt{3}} = \sqrt{\dfrac{21}{3}} = \sqrt{7}$
5) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$4^{1/2} = \sqrt[2]{4^1} = 2^1 = 2$ $8^{4/3} = \sqrt[3]{8^4} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

Note: Radicals can only be added or subtracted if they have the same index and radicand (value underneath the radical).

For example, $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}-5$ CANNOT be simplified.
However the following can be simplified: $5\sqrt{2}-4\sqrt{2}=1\sqrt{2}=\sqrt{2}$

Ex. 13: Simplify. Assume variables can represent any real numbers.

a) $25^{1/2} = \sqrt[2]{25^1}$
$= 5$

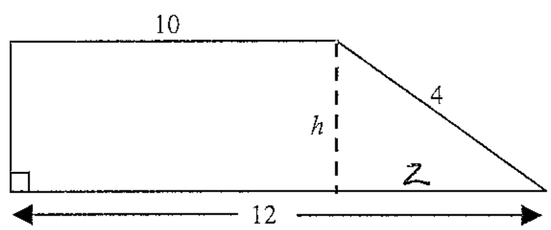
b) $8^{2/3} = \left(\sqrt[3]{8}\right)^2$
$= 2^2 = 4$

c) $\sqrt{50} = \sqrt{25\cdot 2}$
$= 5\sqrt{\cancel{25}\cdot 2}$
$= 5\sqrt{2}$

d) $\dfrac{2\pm\sqrt{12}}{4} = \dfrac{2\pm 2\sqrt{3}}{4}$
$= \dfrac{1\pm\sqrt{3}}{2}$

e) $\left(\sqrt{x}-4\right)\left(\sqrt{x}+4\right)$ ← "conjugates"
$= (\sqrt{x})^2 - 16$
$= x - 16$

Ex. 14: Find the area of the given trapezoid.
$A = \dfrac{1}{2}h(b_1+b_2)$.



Start by finding h
using pyth Thm
$h^2 + 2^2 = 4^2$
$h^2 = 12$
$h = \sqrt{12} = 2\sqrt{3}$
$A = \frac{1}{2}(2\sqrt{3})(12+10)$
$= 22\sqrt{3}$