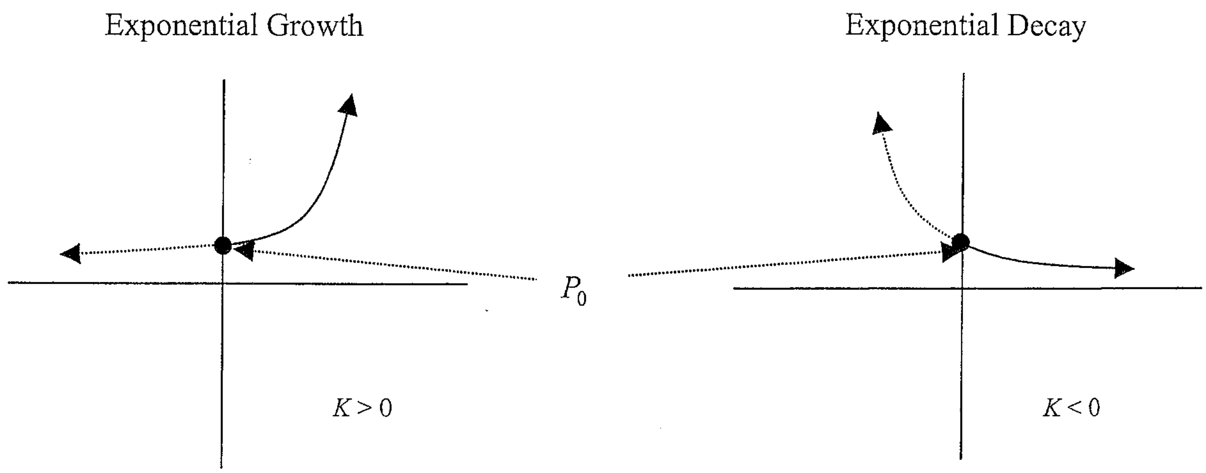
Sec. 5.6 – Applications and Models: Growth and Decay

There are several exponential functions that model real life situations, i.e. applications where a quantity is growing (or decaying) continuously.



$$P(t) = P_0\, e^{kt}$$

$P_0 =$ initial population or amount.
$t =$ time
$k =$ growth/decay rate

Ex. 1: The number of bacteria in a certain culture increased from 600 to 1800 between 7:00 a.m. and 11:00 a.m. Assuming that the growth model is exponential, the number f(t) of bacterial t hours after 7:00 a.m. is given by $f(t) = 600(3)^{t/2}$. Find the number of bacteria at 7:00 a.m. 9:00 a.m. and 11:00 a.m.

$$\boxed{\begin{aligned} &7:00\text{ a.m.} \Rightarrow f(0) = 600(3)^{\circ} = 600(1) = 600 \\ &9:00\text{ a.m.} \Rightarrow f(2) = 600(3)^1 = 1800 \\ &11:00\text{ a.m.} \Rightarrow f(4) = 600(3)^2 = 5400 \end{aligned}}$$

Ex. 2: In 2007, the population of Norway was approximately 4.7 million. Using an annual growth rate of 0.34%, answer the following: $P_0 = 4.7 \quad K = .0034$

a) Find the exponential growth function.

$$P(t) = 4.7e^{.0034t}$$

b) What will the population be in 2012?

$$P(5) = 4.7e^{.0034(5)} \approx 4.78 \text{ mil.}$$

c) Find the amount of time it would take the population to double. This is called the doubling time.

Find the $\underline{t}$ such that P becomes twice P_0.

$$9.4 = 4.7e^{.0034t}$$

$$2 = e^{.0034t}$$

$$\ln 2 = .0034t$$

$$\frac{\ln 2}{.0034} = t$$

$$t \approx 204 \text{ years}$$

NOTE: The half-life of a substance is the time it takes for the amount of a substance to be ½ of what it was initially, i.e. the time it takes for A_0 to become $\frac{1}{2}A_0$.

Ex. 3: Strontium 90 is a radioactive material that decays according to the equation $A = A_0e^{-0.0244t}$, where A_0 is the initial amount present and A is the amount present at time t (in years). If the initial amount is 100 grams, what is the half-life of Strontium 90?

Given $A_o = 100$, find the time it takes for A to reach 50?

$$50 = 100e^{-0.0244t}$$

$$\frac{1}{2} = e^{-.0244t}$$

$$\ell n\frac{1}{2} = -.0244t$$

$$t = \frac{\ell n\frac{1}{2}}{-.0244} \approx 28.4 yrs$$

Ex. 4: Suppose that the cost of a college education can be modeled using exponential growth. In 1995, an average student at ABC University spent \$70,000 for 4 years. In 2005, that same student would spend \$85,000. If the current growth rate continues, what will a 4-year education cost when a new student enrolls in 2015?

Use $P = P_0 e^{kt}$ with $P_0 = 70{,}000$ and $t=0$ corresponding to 1995

Using $P = 85{,}000$ when $t = 10$ allows you to solve for k.

$$85{,}000 = 70{,}000\, e^{10k}$$

$$\frac{17}{14} \rightarrow \frac{85}{70} = e^{10k}$$

$$\ln\left(\frac{85}{70}\right) = 10k$$

$$\frac{1}{10}\ln\left(\frac{85}{70}\right) = k$$

B. Find $P(20)$

$$P = 70{,}000\, e^{20\left(\frac{1}{10}\ln\frac{85}{70}\right)} \rightarrow e^{2\ln\frac{85}{70}}$$

$$\approx \$103{,}214$$

Ex. 5: In the spring, Lake Ruff Ruff is stocked with fish. Due to environmental conditions, the fish population is limited to 3000. The population of fish in the lake after time t, in months, is given by

$$P(t) = \frac{3000}{1 + 4.0e^{-0.28t}}.$$

Note: This type of function (with limited growth) is called a logistic function.

a) What is the initial population?

$$P(0) = \frac{3000}{1+4e^0} = \frac{3000}{1+4(1)} = \frac{3000}{5} = 600$$

b) How long will it take for the population to grow to 2000 fish?

Solve $P(t) = 2000$ for t

$$\frac{3000}{1+4e^{-.28t}} = 2000$$

$$3000 = 2000\left(1+4e^{-.28t}\right)$$

$$3000 = 2000 + 8000\, e^{-.28t}$$

$$1000 = 8000\, e^{-.28t}$$

$$\frac{1}{8} = e^{-.28t}$$

$$t = \frac{\ln\left(\frac{1}{8}\right)}{-.28}$$