1. Find the following limits, if they exist. If the limit does not exist, write "does not exist."

   a) $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x - 3}$

   b) $\lim_{x\to 2} \dfrac{x - 2}{x^2 + 4}$

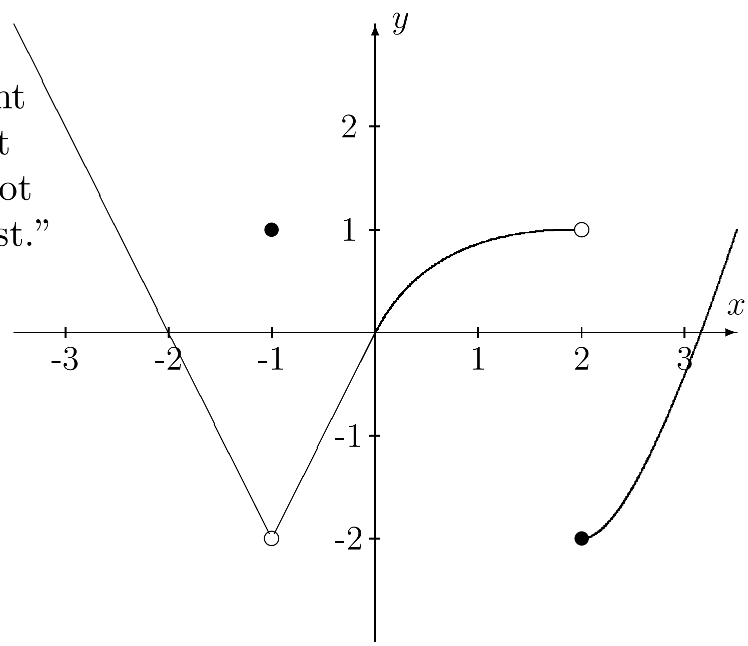
   c) $\lim_{x\to \infty} \dfrac{5x^3 + 14}{7x^3 + 10x}$

2. Use the graph of f at right to estimate each limit if it exists. If the limit does not exist, write "does not exist."

   a) $\lim_{x\to -1} f(x)$

   b) $\lim_{x\to 2} f(x)$

   c) $\lim_{x\to 2^-} f(x)$



3. Find the derivative of each function. Do not simplify.

   a) $f(x) = 3x^5 + e^{-2x}$

   b) $f(x) = (x^3 + 1)^2$

   c) $f(x) = \dfrac{1}{\sqrt{3 + x^2}}$

   d) $f(x) = \dfrac{x^2 + 5x}{x^2 - 4}$

   e) $f(x) = 5 + x^3 \ln x$

Answers

4. Find the second derivative of the function $f(x) = xe^{-x^2}$.

5. Find an equation of the line tangent to $y = 4 + \dfrac{2}{x}$ at $(1, 6)$.

6. Suppose that the monthly demand for the Acme Corporation premium watch is given by the demand equation

$$p = \frac{400}{0.01x^2 + 1}$$

where p denotes the unit price in dollars and x denotes the quantity demanded (in hundreds). How many watches should be made and sold each month in order to maximize the manufacturer's revenue? (Hint: revenue is price times quantity.)

7. Suppose $f(x) = \dfrac{3x^2 + x}{x^2 - 4}$.

Answers

   a) Find all vertical asymptotes for the graph of f.

   b) Find all horizontal asymptotes for the graph of f.

8. Let $f(x) = \dfrac{x^2 + 2x - 3}{x^2}$.

   Then $f'(x) = \dfrac{-2x + 6}{x^3}$ and $f''(x) = \dfrac{4x - 18}{x^4}$.

   a) Find the interval(s) on which $f(x)$ is increasing.

   b) Find all points where relative maxima and minima occur.

   c) Find the interval(s) on which $f(x)$ is concave up.

   d) Find all inflection points.

9. Find the absolute minimum and maximum of the function $f(x) = x^4 - 2x^3$ on the interval $[-1, 3]$.

10. The XYZ Company estimates that the marginal profit from producing and selling x big screen TVs is

$$P'(x) = -0.003x^2 + 0.4x + 200$$

Answers

dollars. What is the difference in profit between making and selling 200 TVs and making and selling 300 TVs?

11. Compute the following integrals:

a) $\int \left(x^3 + 2 + \frac{5}{x^2} \right) dx$

b) $\int xe^{3x}\, dx$

c) $\int_0^1 \frac{e^x}{1+e^x}\, dx$

d) $\int_0^4 \frac{1}{\sqrt{2x+1}}\, dx$

12. Find the present value of the income stream generated at the rate of $R(t) = 10,000$ dollars per year for the next four years if the prevailing interest rate is 6% per year, compounded continuously.

13. Suppose $f(x, y) = x \ln y + y \ln x + \frac{y}{x}$. Find:

a) f_x

b) f_{xy}

14. Let $f(x, y) = x^3 - 3xy + y^3 + 4$. The critical points for this function are $(0, 0)$ and $(1, 1)$. Classify each critical point as a relative maximum, a relative minimum, or a saddle point.

15. Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 16$.

Answers

1. a) 5
   b) 0
   c) $\dfrac{5}{7}$

2. a) -2
   b) does not exist
   c) 1

3. a) $f'(x) = 15x^4 - 2e^{-2x}$
   b) $f'(x) = 2(x^3 + 1) \cdot 3x^2$
   c) $f'(x) = -\frac{1}{2}(3 + x^2)^{-3/2} \cdot 2x$
   d) $f'(x) = \dfrac{(2x + 5)(x^2 - 4) - (x^2 + 5x)(2x)}{(x^2 - 4)^2}$
   e) $f(x) = 3x^2 \ln x + x^3 \cdot \dfrac{1}{x}$

Return to Page 1

4. $f''(x) = (4x^3 - 6x)e^{-x^2}$

5. $y = -2x + 8$

6. Revenue is maximized when 1000 watches are made per month $(x = 10)$.

7. a) $x = 2$ and $x = -2$
   b) $y = 3$

8. a) $(0, 3)$
   b) Relative maximum at $(3, 4/3)$
   c) $(9/2, \infty)$
   d) $(9/2, 35/27)$

Return to Page 2

9. Absolute minimum is $f(3/2) = -27/16$ and absolute maximum is $f(3) = 27$.

10. \$11,000

Return to Page 2

11. a) $\dfrac{x^4}{4} + 2x - \dfrac{5}{x} + C$

    b) $\dfrac{1}{3}xe^{3x} - \dfrac{1}{9}e^{3x} + C$

    c) $\ln \dfrac{1+e}{2}$

    d) 2

12. $\dfrac{10{,}000}{0.06}(1 - e^{-0.24})$

13. a) $f_x = \ln y + \dfrac{y}{x} - \dfrac{y}{x^2}$

    b) $f_{xy} = \dfrac{1}{y} + \dfrac{1}{x} - \dfrac{1}{x^2}$

14. Saddle point at $(0, 0)$ and relative minimum at $(1, 1)$.

15. The maximum value is $f(\frac{4\sqrt{6}}{3}, \frac{4\sqrt{3}}{3}) = \frac{128\sqrt{3}}{9}$.

Return to Page 3