[10]1. Find an equation of the tangent line to the curve of $f(x) = x^3 - 8$, at the point where the curve crosses the x-axis.

[30] 2. Find $\frac{dy}{dx}$ for the following functions. Do not simplify your answers.

a) $y = \frac{\tan x}{x}$

b) $y = x^4 + \frac{1}{x^2}$

c) $y = e^{\cos x} + \sec\left(\frac{x^2}{2}\right)$

d) $y = x^2\sqrt{x+1}$

e) $y = (\sin x)^{x^2}$

f) $y = \int_3^{x^3} e^{t^3}\,dt$

[10]3. Use the definition of the derivative to find $f'(x)$ for $f(x) = \frac{1}{3x}$.

[5] 4. Find $\frac{dy}{dx}$ when $x + y^2 = \cos(xy)$.

[20] 5. Find the following limits. Show all your work.

a) $\lim_{x\to 0} \frac{\sin 9x}{x}$

b) $\lim_{x\to 4} \frac{x^2 - 6x + 8}{x^2 - 16}$

c) $\lim_{h\to 0} \frac{\cos(h) - 1}{h}$

d) $\lim_{x\to\infty} \frac{3x^2 - 1}{3x^2 - x}$

[15] 6. Evaluate the following definite integrals.

a) $\int_{-4}^{4} \sin^5 x\ dx$ b) $\int_0^4 |x - 2|dx$ c) $\int_1^e \frac{(\ln x)^4}{x}dx$

Solutions

[10] 7. Find the absolute maximum and absolute minimum of $f(x) = xe^x + 7$, on $[-2, 0]$.

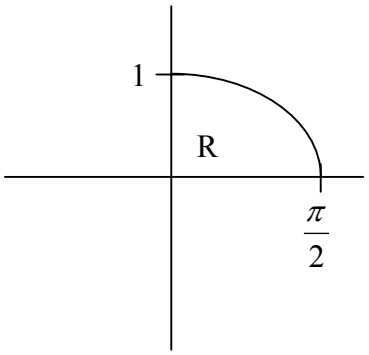
[25] 8. Evaluate the following indefinite integrals.

a) $\int x\sqrt{x-4}\,dx$ b) $\int \frac{\cos x}{\sin^9 x}dx$ c) $\int x^3 e^{x^4}dx$

d) $\int (2x^2 + 2)^2\,dx$ e) $\int \csc^2 x\,dx$

Solutions

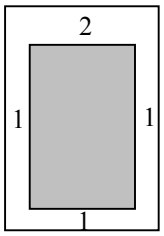
[10]9. Let R be the region bounded by the curves $y=\sqrt{\cos x},\ y=0,\ x=0,\ x=\frac{\pi}{2}$. Find the volume of the solid obtained by revolving this region around the *x*-axis.



[10] 10. Let R be the region bounded by the curves $y=x^2+4,\ y=4,\ x=0,\ x=2$. Find the volume of the solid obtained by revolving this region about the line $x=4$.

[10] 11. Air is being pumped into a spherical balloon and the volume is increasing at the rate of $100\frac{cm^3}{\text{sec}}$. How fast is the radius of the balloon increasing when the radius is 10 cm? $\left(V=\frac{4}{3}\pi r^3\right)$

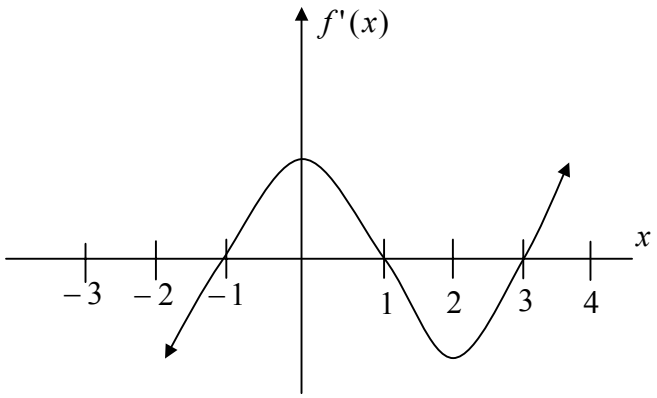
[10]12. A poster is to have an area of $54\,\text{in}^2$ with 1 inch margins at the bottom and sides and a 2 inch margin at the top. What dimensions will give the largest printed area?



Solutions

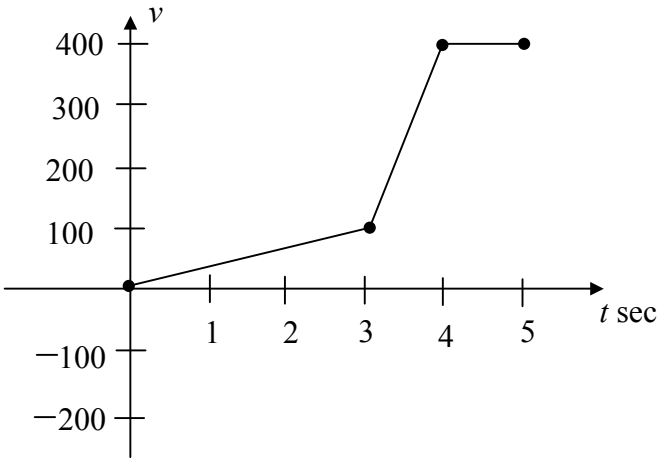
[10] 13. Find the area of the region bounded by the graphs of $y=2x$ and $y=3-x^2$. Also, sketch this region.

[15] 14. Suppose that the DERIVATIVE f' of a function f has the graph



a) Find the open intervals where f is increasing.
b) Find the open intervals where f is concave down.
c) Find the x-coordinates for relative maxima and minima for f. Clearly state which are maximum and which are minimum.

[10] 15. The velocity function (in feet per second) for a particle moving along a line is given by



Find the total distance traveled on the time interval [0,5].

Solutions

1. $f'(x) = 3x^2 \;/\; f(x) = 0 : x^3 = 8 \Leftrightarrow x = 2$

$f'(2) = 12$

$y - 0 = 12(x - 2)$

$y = 12x - 24$

2. a) $\dfrac{dy}{dx} = \dfrac{x\sec^2 x - \tan x}{x^2} \sim= -1$

b) $\dfrac{dy}{dx} = 4x^3 - 2x^{-3}$

$= x^4 + x^{-2}$

c) $\dfrac{dy}{dx} = (-\sin x)e^{\cos x} + x\sec\left(\dfrac{x^2}{2}\right)\tan\left(\dfrac{x^2}{2}\right)$

d) $\dfrac{dy}{dx} = x^2\left(\dfrac{1}{2}\right)(x+1)^{-1/2} + 2x(x+1)^{1/2}$

$= x^2(x+1)^{1/2}$

e) $\ln y = x^2 \ln(\sin x)$

$\dfrac{y'}{y} = x^2\dfrac{\cos x}{\sin x} + 2x\ln(\sin x)$

$y' = y\left\{x^2\dfrac{\cos x}{\sin x} + 2x\ln(\sin x)\right\}$

f) $\dfrac{dy}{dx} = e^{(x^3)^3}\cdot 3x^2$

$= e^{x^9}\cdot 3x^2$

3. $f'(x) = \lim_{h\to 0}\dfrac{\dfrac{1}{3(x+h)} - \dfrac{1}{3x}}{h}$

$= \lim_{h\to 0}\dfrac{\dfrac{3x - 3(x+h)}{3(x+h)3x}}{h} = \lim_{h\to 0}\dfrac{\dfrac{-3h}{9(x+h)x}}{h}$

$= \lim_{h\to 0}\dfrac{-1}{3(x+h)x} = \dfrac{-1}{3(x+0)x} = \dfrac{-1}{3x^2}$

4. $y' = \dfrac{y\sin(xy) + 1}{2y + x\sin(xy)}$

5. a) 9 b) $-\dfrac{1}{4}$ c) 0 d) 1

6. a) 0 b) 4 c) $\dfrac{1}{5}$

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7. Max = 7 at $x = 0$

Min $= 7 - \dfrac{1}{e}$ at $x = -1$

8. a) $\frac{2}{5}(x-4)^{5/2}+\frac{8}{3}(x-4)^{3/2}+C$

b) $-\frac{1}{8\sin^8 x}+C$

c) $\frac{1}{4}e^{x^4}+C$

d) $\frac{4}{5}x^5+\frac{8}{3}x^3+4x+C$

e) $-\cot x+C$

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9. π

10. $\frac{40\pi}{3}$

11. $\frac{1}{4\pi}$

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12. 6×9 outside
4×6 printed

13. $\frac{32}{3}$

14. a) $(-1,1), (3,\infty)$ b) $(0,2)$ c) Rel. Max: $x = 1$
Rel. Min: $x = -1, 3$

15. 800 ft.

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