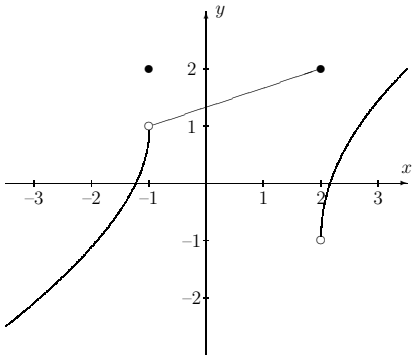
Final Exam–May 10, 2010
Math 140

1. Find the derivative of each function. Do not simplify.
   a) $f(x) = 7x^{-3/2} + 9x^5 + 3$
   b) $f(x) = \sqrt{9 - x^3}$
   c) $f(x) = x^3 e^{-x^2}$
   d) $f(x) = \dfrac{7x - 5}{x^2 + 4}$
   e) $f(x) = x^3 \ln(x + 4)$

2. Use the graph of f at right to estimate each limit if it exists. If the limit does not exist, write "does not exist."
   a) $\lim_{x \to -1} f(x)$
   b) $\lim_{x \to 2} f(x)$
   c) $\lim_{x \to 2^+} f(x)$



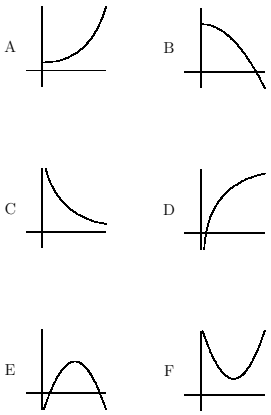
3. Find the following limits, if they exist. If the limit does not exist, write "does not exist."
   a) $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x^2 - x}$
   b) $\lim_{x \to 0} \dfrac{x^2 + 2x - 3}{x^2 - x}$
   c) $\lim_{x \to \infty} \dfrac{7x^2 + 3}{11x^2 - 9x}$

4. For each description below, select the graph at right which matches.

___ a) $f'(x) < 0$ on $(0, \infty)$, $f''(x) < 0$ on $(0, \infty)$

___ b) $f'(x) > 0$ on $(0, \infty)$, $f''(x) > 0$ on $(0, \infty)$

___ c) $f'(x) > 0$ on $(0, \infty)$, $f''(x) < 0$ on $(0, \infty)$



Answers

5. Find the second derivative of the function $f(x) = \dfrac{x+5}{\sqrt{x}}$.

6. Let $f(x) = \dfrac{-3x}{x-1}$.
   a) Find the interval(s) on which $f(x)$ is increasing.
   b) Find the interval(s) on which $f(x)$ is concave up.
   c) Find all vertical asymptotes.
   d) Find all horizontal asymptotes.
   e) Sketch the graph of $y = f(x)$ and label all asymptotes on your graph.

Answers

7. Find the absolute minimum and maximum values of the function $f(x) = x^3 - 2x^2 - 4x + 4$ on the interval $[0, 3]$.

8. Suppose that the weekly demand for a certain model of cell phone is given by the demand equation $p = 180 - 0.3x - 0.02x^2$ where p denotes the unit price in dollars and x denotes the quantity demanded. Find the level of production which maximizes the manufacturer's revenue. (Hint: revenue is quantity times price.)

9. Compute the following integrals:
   a) $\displaystyle\int \left(e^x + 3 + \frac{2}{x}\right) dx$
   b) $\displaystyle\int x^3 \ln x \, dx$ (Use integration by parts.)
   c) $\displaystyle\int_0^2 x^2 \sqrt{x^3+1} \, dx$
   d) $\displaystyle\int_0^2 x(x^2+1)^2 \, dx$

10. Find the area of the region enclosed by the graphs of $f(x) = x^2 - 3x$ and $g(x) = 3 - x$.

11. Find an equation of the line tangent to $y = \dfrac{4}{x+1}$ at $(1, 2)$.

12. Suppose that an investment is expected to generate income at the rate of $R(t) = t+1$ thousand dollars per year for the next four years. Find the present value of the income stream from this investment if the prevailing interest rate is 5% per year compounded continuously.

13. Suppose $f(x, y) = e^{xy} + x^3y^2 - 5x^2$. Find:
   a) f_y
   b) f_{yx}

Answers

14. Let $f(x, y) = x^3 - 6xy - 3y^2$. The critical points for this function are $(0, 0)$ and $(-2, 2)$. Classify each critical point as a relative maximum, a relative minimum, or a saddle point.

15. Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = xy$, subject to the constraint $x^2 + 4y^2 = 2$.

1. a) $-\frac{21}{2}x^{-5/2} + 45x^4$

   b) $\frac{1}{2}(9 - x^3)^{-1/2}(-3x^2)$

   c) $3x^2e^{-x^2} + x^3e^{-x^2} \cdot (-2x)$

   d) $\dfrac{7(x^2+4) - 2x(7x-5)}{(x^2+4)^2}$

   e) $3x^2 \ln(x+4) + \dfrac{x^3}{x+4}$

2. a) 1

   b) does not exist

   c) -1

3. a) 4

   b) does not exist

   c) 7/11

4. a) B

   b) A

   c) D

5. $f''(x) = -\frac{1}{4}x^{-3/2} + \frac{15}{4}x^{-5/2}$

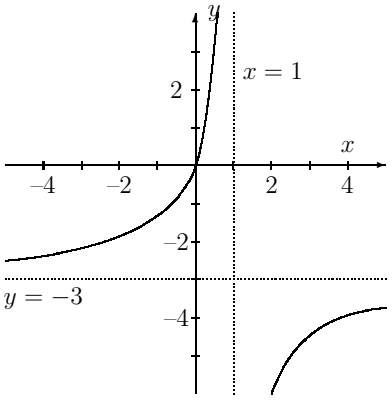
6. a) $(-\infty, 1)$ and $(1, \infty)$

   b) $(-\infty, 1)$

   c) $x = 1$

   d) $y = -3$

   e)



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7. Absolute minimum is $f(2) = -4$ and absolute maximum is $f(0) = 4$.

8. Revenue is maximized when 50 cell phones are produced per week ($x = 50$).

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9. a) $e^x + 3x + 2\ln|x| + C$
   b) $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$
   c) 52/9
   d) 62/3

10. 32/3

11. $y = -x + 3$

12. $420 - 500e^{-0.2} \approx 10.635$ thousand dollars

13. a) $f_y = x\,e^{xy} + 2x^3y$
    b) $f_{yx} = e^{xy} + xy\,e^{xy} + 6x^2y$

14. Saddle point at $(0, 0)$ and relative maximum at $(-2, 2)$.

15. The maximum value is $f(1, \frac{1}{2}) = f(-1, -\frac{1}{2}) = \frac{1}{2}$.