Math108 - Final Exam
May 9, 2011

Spring 2011

Problem	Possible	Earned
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
11	4	
12	4	
13	12	
14	12	
15	12	
16	12	
17	12	
18	12	
19	12	
20	12	
21	12	
22	12	
23-28 **Choose only 4.**	8	
	8	
	8	
	8	
	8	
	8	
Total	200	

Name: Key

Instructor: _____

Section: _____

Please check to make sure that your copy of the examination has all ten (10) pages and a cover sheet with problems numbered 1-28.

Work in a neat and well-organized manner. Show your work on all problems. Indicate your answers clearly.

Calculators will not be allowed for this exam. The use of books, notes or other resource materials will not be permitted on the final examination.

All cell phones and electronic devices are PROHIBITED during the final exam.

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Part I. Problems in this section are mostly short answer and multiple choice. Little partial credit will be given. 4 points each.

1. Express in interval form.
   a) $\{x \mid x < 3\}$ $(-\infty, 3)$
   b) $\{x \mid x \neq -1\}$ $(-\infty, -1) \cup (-1, \infty)$

2. Find the quotient and remainder. $(x^3 - 3x^2 + 5x + 4) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 5 & 4 \\ & & 2 & -2 & 6 \\ \hline & 1 & -1 & 3 & 10 \end{array}$$

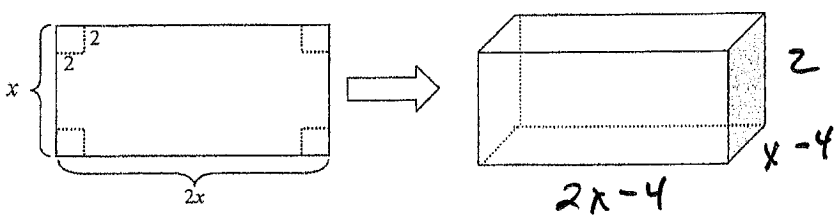
$Q(x) = x^2 - x + 3$; $R(x) = 10$

3. Simplify $(1-2i)^2$ to the form $a+bi$.

$1 - 4i + 4i^2$
$= 1 - 4i - 4$
$= -3 - 4i$

4. An open box is made from a piece of cardboard by cutting a square of side 2 cm from each corner and folding up the edges. If the length of the box (in cm) is twice the width, find the volume of the box in terms of x.

   a) $v(x) = 2x^2 - 16$
   b) $v(x) = 4(2x^2)$
   c) $v(x) = (2x-4)(x-4)$
   d) $v(x) = 2(2x-4)(x-4)$



5. Let $f(x)=3x-1$ and $g(x)=x^2+2$. Find and simplify

a) $(f+g)(x)$

$= 3x-1+x^2+2$

$= x^2+3x+1$

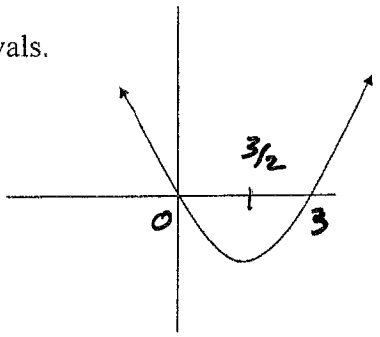
b) $(g\circ f)(-1)$.

$f(-1) = -4$

$g(-4) = (-4)^2+2$

$= 18$

6. Given the graph $f(x)=x^2-3x$, state the following using intervals.



a) All x such that $f(x)$ is decreasing.

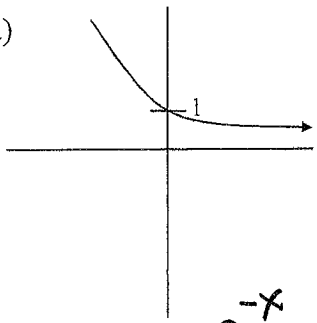
$(-\infty, 3/2)$

b) All x such that $f(x)<0$.

$(0, 3)$

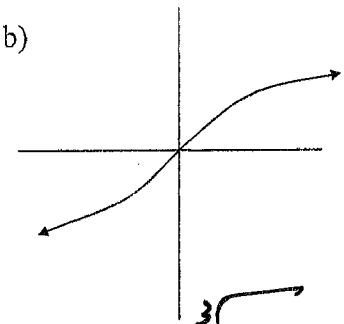
7. From the list of functions, identify each graph.

a)



$f(x) =$ e^{-x}

b)

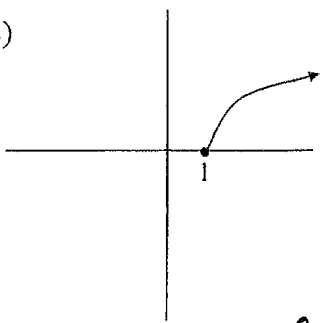


$g(x) =$ $\sqrt[3]{x}$

Choose from the following:

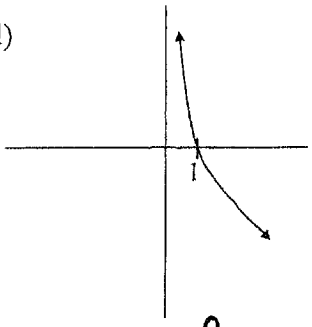
- $\sqrt{x}+1$
- $\sqrt{x+1}$
- $\sqrt[3]{x}$
- $-e^x$
- e^{-x}
- $\ln x$
- $-\ln x$
- Not listed.

c)



$h(x) =$ NOT listed

d)



$m(x) =$ $-\ln x$

8. State the center and radius of the given circle: $(x-2)^2 + y^2 = 9$

center: $(2, 0)$ radius: $r = 3$

9. Solve $A = 2(lw + wh + lh)$ for h.

$A = 2lw + 2wh + 2lh$

$A - 2lw = h(2w + 2l)$

$\frac{A - 2lw}{2w + 2l} = h$

10. Solve: $|2x - 3| > 5$.

$2x - 3 = 5$, $2x - 3 = -5$

$x = 4$, $x = -1$

$x < -1 \cup x > 4$

or $(-\infty, -1) \cup (4, \infty)$

11. State the inverse of each.

a) $f(x) = \sqrt{x+2}$

$f^{-1}(x) = x^2 - 2$

$x = \sqrt{y+2}$

$x^2 = y + 2$

$x^2 - 2 = y$

b) $g(x) = 3x - 4$

$g^{-1}(x) = \frac{x+4}{3}$

$x = 3y - 4$

$\frac{x+4}{3} = y$

12. Evaluate each.

a) $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 4$

b) $\left(\frac{2}{5}\right)^{-2} = \frac{25}{4}$

c) $\log(.001) = -3$

d) $\ln e^{\sqrt{2}} = \sqrt{2}$

Battered, but not Beaten!

Part II. There are 10 problems in this section. Partial credit will be awarded. Show all work. 12 pts. each.

13. Find a linear function h given $h(-2) = 4$ and $h(2) = -3$.

pts: $(-2, 4)$ $(2, -3)$

$m = \frac{4+3}{-2-2}$

$= -\frac{7}{4}$

pt/slope formula

$y - 4 = -\frac{7}{4}(x+2)$

$y = -\frac{7}{4}x - \frac{7}{2} + 4$

$= -\frac{7}{4}x + \frac{1}{2}$

The linear function is $h(x) =$ $-\frac{7}{4}x + \frac{1}{2}$.

14. Solve: $x^{2/3} - 5x^{1/3} + 6 = 0$

$(x^{1/3} - 3)(x^{1/3} - 2)$

$x^{1/3} = 3$ $\quad$ $x^{1/3} = 2$

$x = 3^3$ $\quad$ $x = 2^3$

$= 27$ $\quad$ $= 8$

Solution(s): $x =$ 8, 27.

15. For the function below, find the vertex, zeros, and the maximum or minimum value.

$f(x) = -x^2 + 6x + 7$

zeros: $x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$x = -\frac{6}{-2} = 3$

$f(3) = -9 + 18 + 7$

$= 16$

a) The vertex is (3 , 16).

b) The zeros of f are 7, -1.

c) ~~Minimum~~/maximum value = 16.

16. Solve for x:

a) $2^{3x} \cdot 4^x = \frac{1}{8}$

$2^{3x} \cdot (2^2)^x = 2^{-3}$

$2^{3x} \cdot 2^{2x} = 2^{-3}$

$5x = -3$

$x = -3/5$

b) $\ln(2x-1) = 2$

$e^2 = 2x - 1$

$e^2 + 1 = 2x$

$\frac{e^2+1}{2} = x$

17. Given the polynomial function $f(x) = -(x+3)(2x-3)^2$.

a) State all zeros and their multiplicities.

-3 mult 1

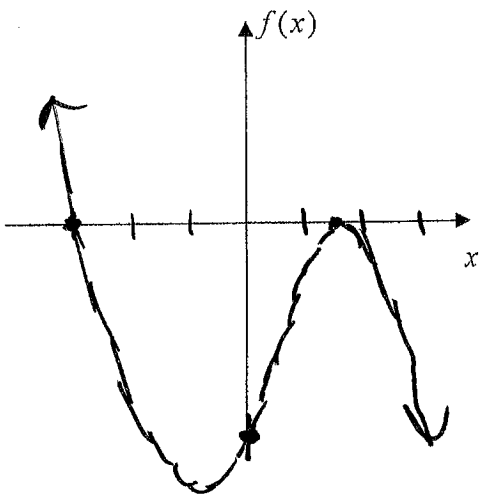
$\frac{3}{2}$ mult 2

Tangent at $x = 3/2$

b) Sketch the graph of $f(x)$. Label all intercepts

$f(0) = -(3)(-3)^2$

$= -27$



18. Solve for x. $x - \frac{12}{x} = 1$

$x^2 - 12 = x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, \; x = -3$

19. Find all asymptotes, x–intercepts, and y–intercepts for the graph of the rational function.

$f(x) = \frac{2x-1}{4-3x}$

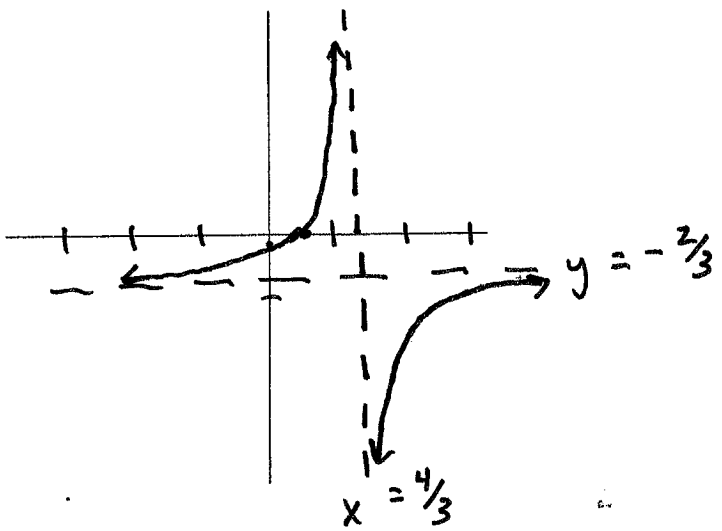
a) The equation of the vertical asymptote(s) is/are $x =$ 4/3.

b) The equation of the horizontal asymptote is $y =$ -2/3.

c) The graph of $f(x)$ has one x–intercept at the point (1/2, 0).

d) The graph of $f(x)$ has one y–intercept at the point (0, -1/4).

e) Sketch the graph $f(x)$. Label all intercepts and asymptotes.



Add pt:

$f(2) = \frac{3}{4-6}$

$= -\frac{3}{2}$

20. Solve for x. $\sqrt{x+7} = x-5$

$x+7 = (x-5)^2$

$x+7 = x^2-10x+25$

$0 = x^2-11x+18$

$0 = (x-9)(x-2)$

$x=9$, ~~$x=2$~~

CHECK

ONLY $x=9$ SATISFIES

$\sqrt{2+7} \neq 2-5$

$\sqrt{9+7} = 7-5$

21. Solve for x. $\log_3(2x-1) - \log_3(x-4) = 2$

$\log_3 \frac{2x-1}{x-4} = 2$

$3^2 = \frac{2x-1}{x-4}$

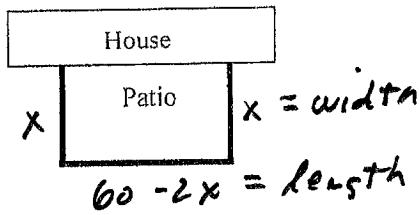
$9(x-4) = 2x-1$

$9x-2x = -1+36$

$7x = 35$

$x = 7$

22. A stone mason has enough stones to enclose a rectangular patio with 60 ft. of stone wall. If a house forms one side of the wall, find



a) Area of the patio as a function of x.

$A(x) = x(60-2x)$

b) Dimensions of patio that yield maximum area.

$A(x) = 60x - 2x^2$

$x = -\frac{b}{2a} = \frac{-60}{-4} = 15$

$l = 60 - 2(15) = 30$

Part III. There are 6 problems in this section. Choose any 4. Indicate in the boxes the problems you want graded. 8 points each.

☐ Grade

23. Given the polynomial $f(x) = x^3 - 8$, find all zeros (real and complex).

$$\begin{array}{r|cccc} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & +8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$f(x) = (x-2)(x^2+2x+4)$

zeros: $x = 2, -1 \pm \sqrt{3}i$

$$x = \frac{-2 \pm \sqrt{-12}}{2(1)}$$

$$= \frac{-2 \pm 2\sqrt{3}i}{2}$$

$$= -1 \pm \sqrt{3}i$$

☐ Grade

24. Find a polynomial function of degree 3 with -1 as a zero of multiplicity 2 and 3 as a zero of multiplicity 1.

$f(x) = (x+1)^2(x-3)$

$= (x^2+2x+1)(x-3)$

$= x^3 - x^2 - 5x - 3$

zero	factor
$x = -1$ mult 2	$(x+1)^2$
$x = 3$	$(x-3)$

$f(x) =$ $x^3 - x^2 - 5x - 3$

☐ Grade

25. The height measured in feet of a rocket t seconds after it has been launched is given by $s(t) = -16t^2 + 72t$, s in feet. Find the time(s) at which the rocket reaches a height of 72 feet.

Solve $-16t^2 + 72t = 72$

$-16t^2 + 72t - 72 = 0$

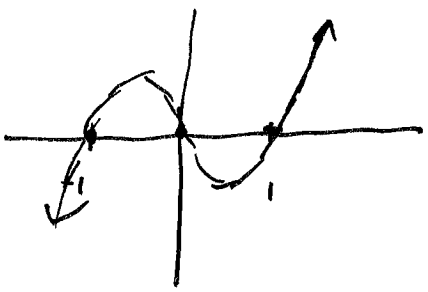
$2t^2 - 9t + 9 = 0$

$(2t - 3)(t - 3) = 0$

$t = 3/2$, $t = 3$

☐ Grade

26. Solve $x^3 - x \geq 0$.



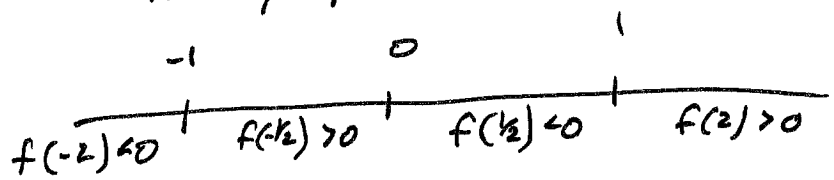
or

solve $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0$

$x = 0, 1, -1$



Sol: $[-1, 0] \cup [1, \infty)$

☐ Grade

27. Solve the system **only for** x. $\begin{cases} x-2y+3z=4 \\ 2x+y-4z=3 \\ -3x+4y-z=-2 \end{cases}$

Elim z:

$x-2y+3z=4$

$-9x+12y-3z=-6$

$-8x+10y=-2$

$2x+y-4z=3$

$12x-16y+4z=8$

$14x-15y=11$

$-24x+30y=-6$

$25x-30y=22$

$4x=16$

$x=4$

☐ Grade

28. Solve for x. $e^{2x}+4e^{x}-5=0$

$u=e^x$

$u^2+4u-5=0$

$(u+5)(u-1)=0$

$u=-5, \quad u=1$

$e^x=-5$

$x=\cancel{\ln}(-5)$

$e^x=1$

$x=\ln 1=0$