SAMPLE EXAM #3 - MATH 150

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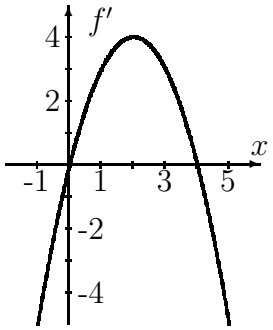
The use of a graphing calculator is not permitted on the test. Show all work.

[12] 1. Let $f(x) = \cos x - x$.
   a) Use the Intermediate Value Theorem to show there is a zero of f between $x = 0$ and $x = \pi/2$.
   b) Use Rolle's Theorem to explain why $f(x)$ has only one zero between $x = 0$ and $x = \pi/2$.

[30] 2. Let $f(x) = \dfrac{x^2 + 6x - 7}{(x+1)^2}$. Then $f'(x) = \dfrac{-4(x-5)}{(x+1)^3}$ and $f''(x) = \dfrac{8(x-8)}{(x+1)^4}$.
   (a) Find the open intervals on which f is increasing and those on which f is decreasing.
   (b) Find the open intervals on which the graph of f is concave up and those on which it is concave down.
   (c) Sketch the graph of f showing and labeling all local extrema, inflection points, and asymptotes.

[20] 3. The graph of $f'(x)$, the *derivative* of $f(x)$, is given at right.
   (a) Find the x-value where f has a local minimum.
   (b) Find the x-value where f has a local maximum.
   (c) Find the x-value where f has an inflection point.



[15] 4. Find the absolute maximum and minimum of $f(x) = 20x^{2/3} - x^{5/3}$ on the interval $[-1, 8]$.

[15] 5. A box with an open top is to be constructed from a square piece of cardboard, 48 cm wide, by cutting out a square from each of the four corners and bending up the sides.
   (a) Write an expression for the volume of the box.
   (b) Find the largest volume that such a box can have. Explain why this value is a maximum.

[8] 6. Find $f(x)$ if $f'(x) = \dfrac{4}{x^2+1} + \dfrac{3}{x^2}$, $x > 0$, and $f(1) = 2$.

Date: Fall 2009.