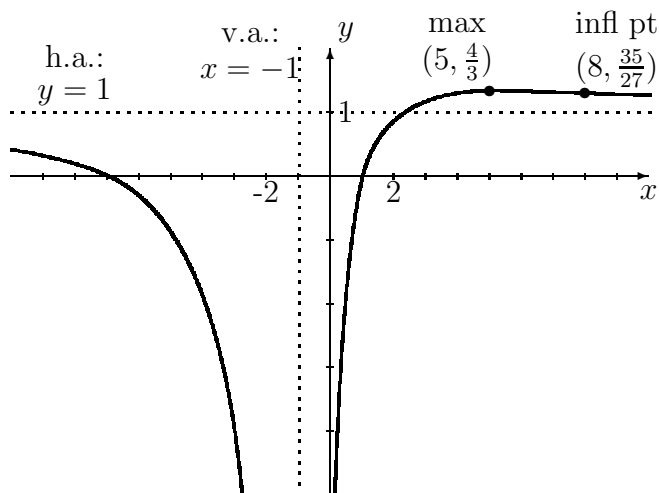


ANSWERS TO SAMPLE EXAM #3 - MATH 150

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Answers are given. To receive full credit on the test the student must show all work.

1. (a) Since f is continuous everywhere, it is continuous on the interval $[0, \pi/2]$. Also, $f(0) = 1 > 0 > -\pi/2 = f(\pi/2)$. Thus, by the Intermediate Value Theorem, there is a $c \in [0, \pi/2]$ such that $f(c) = 0$.
- (b) Suppose there were two zeros between 0 and $\pi/2$, $r_1 < r_2$. Since f is continuous and differentiable everywhere, it is continuous on $[r_1, r_2]$ and differentiable on (r_1, r_2) . Also $f(r_1) = 0 = f(r_2)$. By Rolle's Theorem, there would have to be a $c \in (r_1, r_2) \subset (0, \pi/2)$ such that $f'(c) = 0$. However, $f'(x) = -\sin x - 1 \leq -1$ (never equals 0) on $(0, \pi/2)$. Therefore there cannot be more than one zero on this interval.
2. (a) Increasing on $(-1, 5)$ and decreasing on $(-\infty, -1)$ and $(5, \infty)$
- (b) Concave up on $(8, \infty)$ and concave down on $(-\infty, -1)$ and $(-1, 8)$



- (c)
3. (a) $x = 0$
- (b) $x = 4$
- (c) $x = 2$
4. Absolute maximum is 48 (at $x = 8$) and absolute minimum is 0 (at $x = 0$).
5. (a) Volume = $V = x(48 - 2x)^2$, $0 < x < 24$, where x is the length of a side of a cut out square.
- (b) V is maximized when $x = 8$ cm. The maximum volume is 8192 cm^3 . This value is a maximum since V is a continuous function of x on the interval $[0, 24]$ and $V = 0$ when $x = 0$ or $x = 24$. $x = 8$ is the only critical number inside the interval.
6. $f(x) = 4 \arctan x - \frac{3}{x} + 5 - \pi$