Sample Test 3

1. Matching

____1. Arclength in rectangular, ds

____2. arclength in polar, ds

____3. *dy/dx*

____4. d^2y/dx^2

____5. Rectangular area, dA

____6. Polar area, dA

a. $\dfrac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3}$

b. $\dfrac{\dot{y}}{\dot{x}}$

c. $y\,dx$

d. $y\,d\theta$

e. $\left(r^2 + \left(\dfrac{dr}{d\theta}\right)^2 \right)^{1/2}$

f. $(\dot{x}^2 + \dot{y}^2)^{1/2}\,dt$

g. $\frac{1}{2}\, r^2\, d\theta$

h. $r\,d\theta$

i. x^2+y^2

2. Let $f(x) = \dfrac{1}{1-2x}$

(a) Expand *f* as a Maclaurin series. Show at least three terms and the general term. State the domain of convergence (as an interval).

(b) Expand *f* as a Taylor series around $a = 3$. Show at least three terms and the general term. You may keep out a common factor. State the domain of convergence (as an interval).

(c) Use two terms of the series of part b. to estimate $f(3.25)$.

3. (a) Find the complete Taylor expansion of $f(x) = 2 - x^2$ around $a = 1.4$.

(b) Use the expansion of part a. to evaluate $f(1.41)$ and $f(1.42)$. Which is closer to zero?

(c) Shift the variable x to $x+1.4$ to get the Maclaurin expansion of $f(x+1.4)$, i.e., of $2 - (x + 1.4)^2$.

4. (a) Write four terms of the Maclaurin series for sin x.

(b) Divide both sides of your answer to part (a) by x to get four terms of the Maclaurin series

$$\frac{\sin x}{x} =$$

(c) Differentiate the series (right-hand side) of part (b) to find three terms of the Maclaurin series

$$\left(\frac{\sin x}{x}\right)' =$$

(d) Integrate the result of part b. to get four terms of the expansion of

$$\int_0^1 \frac{\sin x}{x}\,dx =$$

(e) Sum three then four terms of the expansion of part (d) to get upper and lower bounds on the value of the integral. Show results truncated to 6 decimals. How many/which decimals are exact?

5. Identify the polar curve $r = \sqrt{3} - 2\sin\theta$.

Make a sketch — a useful chart includes values of $\theta = 0, \pi/6, \pi/3, \pi/2, 2\pi/3, 5\pi/6, \pi, 3\pi/2, 2\pi$.

Solve $r = 0$ for values of θ in the interval $(0, 2\pi)$.

Find the area of the inner loop: give an exact expression for that area and then evaluate it rounded to two decimals.

Evaluate $r = r_0$ at the lowest point of the inner loop and interpret $|r_0|$ as the diameter of a circle. Find the area of a circle with *diameter* equal to $|r_0|$ as an estimate of your answer for the area of the loop.

EXTRA

6. (a) Determine the eccentricity (absolute value) and identify the conic as one of four types: circle, parabola, hyperbola, ellipse.

(i) $r = \dfrac{3}{2+8\sin\theta}$ (ii) $r = \dfrac{2}{4+\sin\theta}$

(b) Convert $r = 4\sin\theta$ to rectangular coordinates. Identify the curve precisely.

(c) Convert $2x+3y = 1$ to polar coordinates. Simplify to the form $r = f(\theta)$. Identify the curve precisely.