

1. The mean of 3 numbers is 8. Their median is 6. If the smallest number is 3, what is the largest number?

- a) 12            b) 13            c) 14            d) 15            e) 16

2.  $3^{10} + 3^{10} + 3^{10}$  is equal to

- a)  $9^{10}$             b)  $3^{30}$             c)  $3^{11}$             d)  $3^{1000}$             e) None listed.

3. A line with a slope of 2 intersects a line with a slope of 4 at the point (30, 20). What is the distance between  $x$ -intercepts of the lines?

- a) 5            b) 3            c) 7            d) 9            e) 10

4. Which of the following is **NOT** equal to  $1/6$ ?

- a)  $\frac{1}{2} - \frac{1}{3}$             b)  $\frac{1}{15} + \frac{1}{10}$             c)  $\frac{2}{21} + \frac{1}{14}$             d)  $\frac{1}{4} + \frac{1}{12}$             e) None listed.

5. How many of the following numbers are prime?

111            57            221            257

- a) 0            b) 1            c) 2            d) 3            e) 4

6. At night, a man who is 6 feet tall stands several feet away from a lamppost. The lamppost's light bulb is 16 feet above the ground. If the man's shadow is 3 feet long, then how far away is the man from the lamppost?

a)  $\frac{15}{8}$  feet      b) 3 feet      c)  $\frac{10}{3}$  feet      d) 4 feet      e) 5 feet

7. Given  $\sec x - \tan x = 2$ , what is  $\sec x + \tan x$ ?

a) 1      b) 2      c)  $\frac{1}{2}$       d) 0      e) -1

8. For what value of parameter  $m$  does the following equation have exactly one solution?

$$x^2 + 10x + 16 = 2mx + 4m$$

Which group contains your answer?

a)  $0, \frac{8}{5}, \frac{1}{2}$       b)  $\sqrt{2}, \sqrt{3}, 3$       c)  $1, -7, 8$       d)  $-1, 5, 6$       e)  $-3, 2, -\sqrt{2}$

9. Nine chairs in a row are to be occupied by six students and Professors Abel, Bessel, and Cotes. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Able, Bessel, and Cotes choose their chairs?

a) 50      b) 60      c) 70      d) 80      e) 90

10. For what value of  $n$  is  $i + 2i^2 + 3i^3 + \dots + ni^n = 48 + 49i$ ? Note: Here  $i = \sqrt{-1}$ .

a) 24      b) 48      c) 49      d) 97      e) 98

11. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

- a) 21                      b) 30                      c) 27                      d) 18                      e) 24

12. How many of the following are equal to  $\frac{1}{2}$ ?

$2\sin 15^\circ \cos 15^\circ$        $4\cos^4 22.5^\circ + 4\cos^2 22.5^\circ + 1$        $4\cos^3 20^\circ - 3\cos 20^\circ$        $\sin^2 15^\circ$

- a) 0                      b) 1                      c) 2                      d) 3                      e) 4

13. Perimeter of a rectangle is 18 cm. Its diagonal is 8 cm. Find the area of the rectangle. Round to three decimal places.

- a) 10.000              b) Exactly 10              c) 10.003              d) 7.413              e) 8.500

14. How many natural numbers not exceeding 2001 are multiples of 3 or 4 but not 5?

- a) 800                      b) 801                      c) 802                      d) 803                      e) 804

15. Consider an urn with 10 red marbles and 10 blue marbles. For each step of the game, remove one marble from the urn and return it to the urn along with a new marble of the same color. What is the probability of drawing red on the second step?

- a)  $\frac{1}{2}$                       b)  $\frac{11}{21}$                       c)  $\frac{10}{21}$                       d)  $\frac{11}{21} \cdot \frac{1}{2}$                       e)  $\frac{10}{21} + \frac{11}{21}$

16. If  $a < 0$  then  $\left| a - \sqrt{(a-1)^2} \right|$  equals?

- a) 1                      b)  $2a+1$                       c)  $1-2a$                       d)  $-2a-1$                       e)  $2a-1$

17. Let  $x = \left( 3^{\log_3 4} \right)^{\log_4 (2x-2)}$ . Then  $x = ?$

- a) 2                      b) 3                      c) 4                      d) 5                      e) 6

18. In triangle  $ABC$ ,  $3\sin A + 4\cos B = 6$  and  $4\sin B + 3\cos A = 1$ . Then  $C =$

- a)  $15^\circ$                       b)  $30^\circ$                       c)  $45^\circ$                       d)  $60^\circ$                       e)  $75^\circ$

19. In the expansion of  $(1+x+x^2+\dots+x^{27})(1+x+x^2+\dots+x^{14})^2$ , what is the coefficient of  $x^{28}$ ?

- a) 195                      b) 196                      c) 224                      d) 378                      e) 405

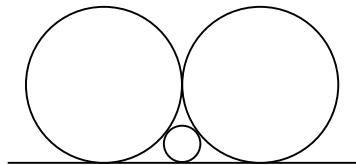
20. Which of the following functions does *not* have an inverse?

- a)  $f(x) = x$                       b)  $f(x) = \frac{1}{x}$                       c)  $f(x) = x^2$                       d)  $f(x) = x^3$                       e)  $f(x) = x^{1/3}$

21. In an arena, each row has 199 seats. One day 1990 students are coming to attend a math contest. It is only known that at the most 39 students are from the same school. (You do not know the number of schools coming or the specific number of students from any one school.) If students from the same school must sit in the same row, what is the minimum number of rows that must be reserved for these students?

a) 12                      b) 13                      c) 14                      d) 15                      e) 16

22. Three cylindrical tubes, two bigger and one smaller, run on the floor, as shown in the figure. The diameter of each bigger tube is 12. What is the diameter of the smaller tube? (Approximate.)



a) 2.98                      b) 3                      c) 4.2                      d)  $2\pi$                       e)  $\sqrt{6}$

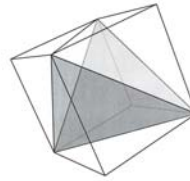
23. Let  $p_1, p_2, p_3, \dots, p_{20}$  be the first twenty prime numbers. Which of the following is certainly true of the number  $p_1 \cdot p_2 \cdot p_3 \cdots p_{20} + 1$ ?

a) It must be even.  
 b) It must be divisible by at least one of  $p_1, p_2, p_3, \dots, p_{20}$ .  
 c) It must be divisible by all  $p_1, p_2, p_3, \dots, p_{20}$   
 d) It may be divisible by some of  $p_1, p_2, p_3, \dots, p_{20}$   
 e) It is not divisible by any of  $p_1, p_2, p_3, \dots, p_{20}$

24. Six couples are to be seated in a row of 12 chairs. In how many ways can they be seated if spouses have to sit next to each other?

a)  $\frac{6!}{(2!)^6}$                       b)  $12!$                       c)  $(2!)(6!)$                       d)  $6!(2!)^6$                       e)  $(12!)(2!)^6$

25. A pyramid is formed by connecting alternate vertices of a cube as shown. If the volume of the cube is 1, what is the volume of the pyramid?



- a)  $\frac{1}{3}$       b)  $\frac{1}{6}$       c)  $\frac{1}{2}$       d)  $\frac{\sqrt{3}}{2}$       e) None listed.
26. Suppose that  $f(x+3) = 3x^2 + 7x + 4$  and  $f(x) = ax^2 + bx + c$ . What is  $a + b + c$ ?
- a) 2      b) 0      c) 3      d) -1      e) 1
27. What is the value of the following product?  $\sin \frac{\pi}{32} \cos \frac{\pi}{32} \cos \frac{\pi}{16} \cos \frac{\pi}{8} \cos \frac{\pi}{4}$
- a)  $\frac{1}{2}$       b)  $\frac{1}{4}$       c)  $\frac{1}{8}$       d)  $\frac{1}{16}$       e)  $\frac{1}{32}$
28. Three numbers:  $2$ ,  $2^{3x-4}$ ,  $2x^2$  are consecutive terms of an increasing geometric sequence. Find  $x$ . In which of these groups is your answer?
- a)  $0, \frac{1}{3}, \frac{1}{2}$       b)  $0.781, \sqrt{2}, \sqrt{3}$       c)  $1, 7, 8$       d)  $-1, -2, \sqrt{2}$       e)  $2, 3, -100$
29. Find the area in the second quadrant bounded by the lines  $x - y + 2 = 0$  and  $x - y + 4 = 0$ .
- a) 2      b) 3      c) 4      d) 5      e) 6

30. For what value of  $x$  does  $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$ ?
- a) 8                      b) 16                      c) 32                      d) 256                      e) 1024
31. A coin is flipped and a die is rolled simultaneously. This experiment is repeated over and over. What is the probability of observing a "six" before observing a "head"?
- a)  $\frac{1}{6}$                       b)  $\left(\frac{1}{6} - \frac{1}{2}\right)$                       c)  $\left(\frac{1}{2} - \frac{1}{6}\right)$                       d)  $\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$                       e)  $\frac{1}{7}$
32. Bob can finish a task in 5 days and Mary can do it in 20 days. Together they can finish it in how many days?
- a) 4                      b) 3                      c) 2                      d) 15                      e) 25
33. Consider a circle with diameter 10. Inscribe a square in the circle; that is, construct a square whose corners all lie on the circle. What is the area of the square?
- a)  $\frac{10}{\sqrt{2}}$                       b)  $\frac{100}{\sqrt{2}}$                       c) 50                      d) 100                      e) Not enough information to determine.
34. If  $x + \frac{1}{x} = 4$ , then  $x^3 + \frac{1}{x^3}$  is equal to
- a) 52                      b) 60                      c) 64                      d) 68                      e) 76
35. Let  $x = \cos 2$ ,  $y = \cos 3$ ,  $z = \cos y$  (angles are in radians). Which of the following is true?
- a)  $x < z < y$                       b)  $x < y < z$                       c)  $y < x < z$                       d)  $y < z < x$                       e)  $z < x < y$

36. How many primes divide  $60 \cdot 57 \cdot 47 \cdot 111 \cdot 91$ ?

- a) 5                      b) 10                      c) 6                      d) 7                      e) 8

37. Suppose  $b$  is a solution to quadratic equation  $x^2 - x - 1 = 0$ . Evaluate  $b^4$ .

- a)  $b+1$                       b)  $2b+1$                       c)  $b+3$                       d)  $3-b$                       e)  $3b+2$

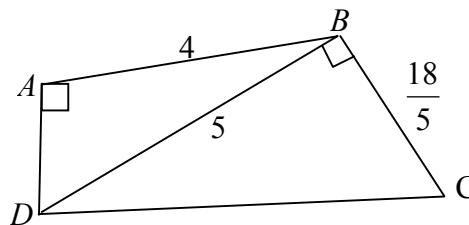
38. From a set of four segments of length 2, 3, 5, and 6, three segments are chosen at random. What is the probability that one may construct with them a triangle of non-zero area?

- a) 1                      b) 0                      c) 0.25                      d) 0.5                      e) 0.3333

39. If  $3\sin\theta + 4\cos\theta = 5$ , then  $\tan\theta$  is equal to

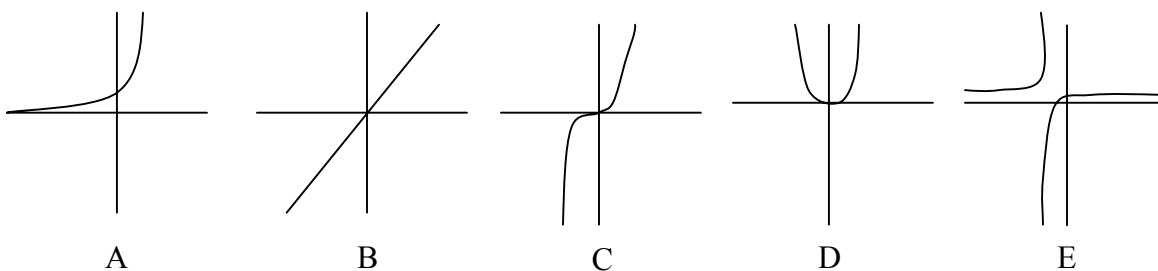
- a) 0                      b)  $\frac{4}{3}$                       c)  $\frac{3}{4}$                       d) 1                      e) -1

40. Find  $AC$ .



- a)  $\frac{1714}{5}$                       b)  $\frac{34}{5}$                       c)  $\frac{28}{5}$                       d) None listed.                      e) Not enough information.

41. Which of the following most closely resembles the graph of  $g(x) = 2x^8 - 7.2x^3$ ?



- a) A                      b) B                      c) C                      d) D                      e) E

42. Two initial consecutive terms of a geometric sequence are 3 and 5. What is the third term?

- a) 7                      b) 15                      c)  $\sqrt{15}$                       d)  $3\sqrt{5}$                       e)  $8\frac{1}{3}$

43. Suppose  $\tan \alpha = \frac{4}{3}$  for some  $\alpha$ ,  $0 < \alpha < 90^\circ$ . Find the exact value of  $\sin \alpha + \cos \alpha$ .

- a) 0                      b) 1                      c)  $\frac{5}{7}$                       d)  $\frac{7}{5}$                       e)  $\frac{1}{3}$

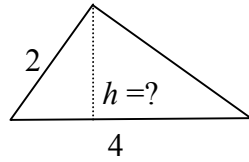
44. Let  $a + ar_1 + ar_1^2 + ar_1^3 + \dots$  and  $a + ar_2 + ar_2^2 + ar_2^3 + \dots$  be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is  $r_1$  and the sum of the second series is  $r_2$ . What is  $r_1 + r_2$ ?

- a) 0                      b) 12                      c) 1                      d)  $\frac{1+\sqrt{5}}{2}$                       e) 2

45. How many ways can the numbers 21, 31, 41, 51, 61, 71, and 81 be arranged such that the sum of every four consecutive numbers is divisible by 3?

- a) 81                      b) 121                      c) 144                      d) 169                      e) 196

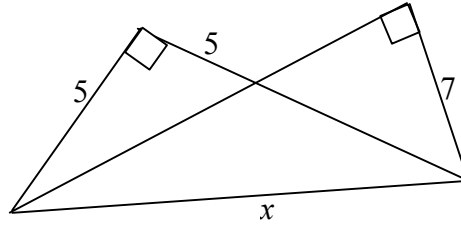
46. What is the height of this right triangle, dropped from the right angle towards the hypotenuse?



- a) 2.4      b) 3.5      c) 3.7      d)  $\sqrt{3}$       e)  $2\sqrt{6}$
47.  $\sqrt{3}$  percent of  $200\sqrt{3}$  is
- a) 0.5      b) 3      c) 600      d) 6      e) None listed.
48. For which real numbers  $a$  does the equation  $x^2 + 2x - a = 0$  have no real solutions?
- a)  $a < -1$       b)  $a \leq -1$       c)  $a > 2$       d)  $a > 1$       e)  $a \geq 1$
- 49.
50. A clock shows slightly after 5:25. More precisely, the two hands are perfectly aligned. What is the exact time (to the nearest tenth of a minute)?
- a) 5:25.7      b) Exactly 5:29      c) 5:27.3      d) 5:27.4      e) 5:28.1

51. A car is running on a circular track. The outer wheels are going twice as fast as the inside ones. If they are 3 feet apart, what is the length of the circumference drawn by the outer wheels?
- a)  $6\pi$                       b)  $12\pi$                       c)  $2\pi$                       d) 3                      e) 6
52. A six-person committee consisting of three married couples must form two three-person subcommittees. By rule, no committee member may serve on both subcommittees and no committee member may serve on the same subcommittee as his or her spouse. In how many ways can the committees be formed?
- a) 2                      b) 6                      c) 8                      d) 12                      e) 0
53.  $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3)$
- a)  $\sin 9^\circ$                       b)  $\cos 9^\circ$                       c)  $\tan 9^\circ$                       d)  $\cot 9^\circ$                       e) 1
54. The increasing sequence 1, 3, 4, 9, 10, 12, 13, ... consists of all the natural numbers which are powers of 3 or sums of distinct powers of 3. What is the 100<sup>th</sup> term in this sequence?
- a) 243                      b) 729                      c) 973                      d) 976                      e) 981
55. A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers  $x$  and  $x + 32$  are three-digit and four-digit palindromes, respectively. What is the sum of the digits of  $x$ ?
- a) 20                      b) 21                      c) 22                      d) 23                      e) 24

56. Find  $x$ .



- a) 13      b)  $6\sqrt{6}$       c)  $\sqrt{148 + 70\sqrt{2}}$       d)  $3\sqrt{11}$       e)  $2\sqrt{37}$

57. Suppose \$1000 is invested at 5% interest, compounded monthly for a period of 50 years. Will the future value of the investment be most nearly?

- a) \$5      b) \$50      c) \$2500      d) \$10,000      e) \$1,000,000

58. Let  $a, b, c, d$ , and  $e$  be positive integers with  $a + b + c + d + e = 2011$ , and let  $M$  be the largest of the sums  $a + b$ ,  $b + c$ ,  $c + d$  and  $d + e$ . What is the smallest possible value of  $M$ ?

- a) 670      b) 671      c) 802      d) 803      e) 804

59.  $\sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$

- a)  $\cos 2x$       b)  $\sin 2x$       c)  $\cos x$       d)  $\sin x$       e) 1

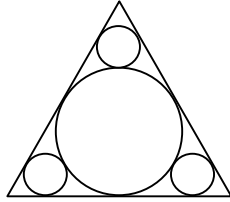
60. What is the number of ways of choosing five numbers from the first 18 natural numbers such that any two chosen numbers differ by at least 2?

- a) 1999      b) 2000      c) 2001      d) 2002      e) 2003

61.  $\cos 72^\circ - \cos 36^\circ =$

- a)  $\frac{1}{3}$       b)  $-\frac{1}{3}$       c)  $\frac{1}{2}$       d)  $-\frac{1}{2}$       e) 1

62. What is the ratio of the areas of the two circles, big to small, in the design based on the equilateral triangle shown here?



- a) 8      b) 9      c) 10      d) 11      e) 12

63. Let  $f(x)$  be a function that satisfies  $f(x + y) = f(xy)$  for all integers  $x$  and  $y$ , and  $f(7) = 5$ . Find the values of  $f(5)$ .

- a) 1      b) 5      c) 7      d) 14      e) None listed.

64. Numbers 2 and  $-1$  are roots of the polynomial  $2x^3 + ax^2 + bx - 4$ . Find the values  $a$  and  $b$ . Their sum,  $a + b$  is

- a)  $-6$       b) 1      c)  $-1$       d) 14      e)  $-3$

65. If  $-1 < a < 0$ , which of the following is true?

a)  $a < a^2 < a^3$

b)  $a < a^3 < a^2$

c)  $a^2 < a < a^3$

d)  $a^2 < a^3 < a$

e)  $a^3 < a < a^2$

66. If  $n$  is positive, which of the following is equal to  $\frac{1}{\sqrt{n+1} - \sqrt{n}}$ ?

a) 1      b)  $\sqrt{2n+1}$       c)  $\frac{\sqrt{n+1}}{\sqrt{n}}$       d)  $\sqrt{n+1} - \sqrt{n}$       e)  $\sqrt{n+1} + \sqrt{n}$

67. Which of the following is equivalent to the pair of inequalities  $x + 6 > 10$  and  $x - 3 \leq 5$ ?

a)  $2 \leq x < 16$

b)  $2 \leq x < 4$

c)  $2 < x \leq 8$

d)  $4 < x \leq 8$

e)  $4 \leq x < 16$