

A Martingale Approach to Asymptotic Stability of Discrete SDE with Bounded Noise in \mathbb{R}^1

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ABSTRACT: Necessary and sufficient conditions for almost sure asymptotic stability of solutions to linear and nonlinear, nonautonomous ordinary stochastic difference equations (SDE)

$$X_{n+1} = X_n (1 - \alpha_n f(X_n) + \sigma_n g(X_n) \xi_{n+1})$$

driven by square-integrable martingale-differences $(\xi_{n+1})_{n \in \mathbb{N}}$ with uniformly bounded quantities $\sigma_n \xi_{n+1}$ in \mathbb{R}^1 are in the center of this presentation. All conditions are explicitly expressed in terms of the coefficients α_n , σ_n , f and g . Kolmogorov's variant of the strong law of large numbers as well as martingale convergence and martingale representation theorems are applied to prove related results.

Key words and phrases. Stochastic differential equations, asymptotic stability, sufficient and necessary conditions, Martingale convergence theorems, Kolmogorov's law of large numbers.

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