

# On an Exponential Martingale Approach to Almost Sure Stability of Itô SDEs in $\mathbb{R}^1$

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**ABSTRACT:** Almost sure stability of solutions to linear and nonlinear, nonautonomous stochastic differential equations (SDEs) driven by a standard Wiener process are discussed. For this purpose, we confine our study to SDEs with trivial equilibrium in  $\mathbb{R}^1$  and exploit the well-known ideas of Dolean-Dade exponentials, martingale convergence and representation theorems, and Khinchin's law of iterated logarithm. Necessary and sufficient conditions of a.s. asymptotic stability of the trivial solution are obtained. All conditions are expressed in terms of coefficients of those equations. Eventually, we discuss polynomial decay rates to classify the asymptotic decay of its solutions.

**Key words and phrases.** Stochastic differential equations, a.s. asymptotic stability, Dolean-Dade exponential, stochastic exponentials, martingale convergence theorems, law of iterated logarithm for stochastic integrals, polynomial decay rates.