

# Rates of Convergence of Approximations of Some Convex Functionals of Stochastic Differential Equations

Preprint m-05-002

Henri Schurz

*Department of Mathematics, Southern Illinois University Carbondale  
Carbondale, IL 62901-4408, USA*

*and*

*Texas Tech University, Department of Mathematics and Statistics,  
Lubbock, TX 79409-1042, USA*

**ABSTRACT:** The rate of convergence of numerical methods for integration of some convex functionals of ordinary stochastic differential equations (SDEs) is discussed. In particular, we answer how rates of  $p$ -th mean convergence carry over to rates of weak convergence for non-smooth and convex functionals of SDEs. Qualitative behavior of some numerical approximations such as non-negativity of balanced implicit Milstein methods is investigated as well. Nonstandard integration techniques seem to be the most promising ones. As a main result we obtain some justification for the choice of approximation schemes of discounted price functionals in dynamic asset pricing.

**Key words and phrases.** Stochastic differential equations, Noisy ODEs, Approximation of convex functionals, Numerical methods, Qualitative behavior, Stability, Convergence, Contractivity, Nonnegativity, Discounted price functionals, Asset pricing, Rate of weak and  $p$ -th mean convergence, Approximation of stochastic exponentials.