Dynamics of Linearly Partial-implicit Midpoint Methods for Numerical Integration of Some Infinite Systems of ODEs with Cubic-type Nonlinearity and Q-regular Additive Noise

Preprint m-05-003

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ABSTRACT: Qualitative aspects of numerical methods for integration of systems of nonlinear ordinary stochastic differential equations (SDEs) with potential applicability to mechanical engineering are presented. In particular, we study the qualitative behavior of some linearly partial-implicit midpoint-type methods for numerical integration of some infinite systems of SDEs with cubic-type nonlinearity and Q-regular additive space-time noise. Construction and properties such as stability and convergence of such stochastic-numerical methods is strongly related to their uniform boundedness along Lyapunov-type functionals. Well-known convergence order bounds apart from further complexity issues forces us to focus our analysis on lower order Runge-Kutta methods rather than higher order Taylor methods. Nonstandard techniques such as partial-implicit difference methods for noisy ODEs/PDEs seem to be the most promising ones in view of adequate long term integration of such nonlinear systems.

Key words and phrases. Stochastic differential equations, Noisy ODEs, Partial-implicit numerical methods, Qualitative behavior, Stability, Convergence, Boundedness, Energy- and Lyapunov-type functionals.