ANALYSIS AND DISCRETIZATION OF SOME QUASI-LINEAR SPDEs WITH CUBIC-TYPE NONLINEARITIES AND Q-REGULAR NOISE

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ABSTRACT: Some quasilinear stochastic partial differential equations (SPDEs) with homogeneous boundary conditions and Q-regular space-time noise are considered (i.e. models of noisy nonlinear string and noisy nonlinear beam). First, we discuss existence and uniqueness of (strong) solutions, and uniform boundedness of some moments using Lyapunov-type functionals. Second, we introduce truncated models leading to systems of ordinary stochastic differential equations (SDEs). Third, partial-implicit difference methods for numerical integration of such stochastic systems are presented. In particular we study their qualitative behavior with respect to the energy functional. Well-known convergence order bounds apart from further complexity issues forces us to focus on lower order partial-implicit Runge-Kutta methods rather than higher order Taylor methods. Nonstandard midpoint-based techniques for noisy PDEs / SDEs are suggested for numerical integration. The results can be used to estimate probabilities of large fluctuations (random vibrations).

Key words. Random PDEs, random vibrations, semilinear stochastic differential equations, stochastic systems, Q-regular noise, existence, uniqueness, boundedness of moments, energy methods, discretization, nonstandard difference methods, partial-implicit midpoint methods, convergence, stability, Lyapunov functionals, probability of large fluctuations.

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