MODELING, ANALYSIS AND DISCRETIZATION OF STOCHASTIC LOGISTIC EQUATIONS

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ABSTRACT: The well–known logistic model has been extensively investigated in deterministic theory. There are numerous case studies where such type of nonlinearities occur in Ecology, Biology and Environmental Sciences. Due to the presence of environmental fluctuations and a lack of precision of measurements, one has to deal with effects of randomness on such models. As a more realistic modeling, we suggest nonlinear stochastic differential equations (SDEs)

$$dX(t) = [(\rho + \lambda X(t))(K - X(t)) - \mu X(t)]dt + \sigma X(t)^{\alpha}|K - X(t)|^{\beta} dW(t)$$

of Itô-type to model the growth of populations or innovations X, driven by a Wiener process W and positive real constants ρ, λ, K, μ, α, $\beta \geq 0$. We discuss well–posedness, regularity (boundedness) and uniqueness of their solutions. However, explicit expressions for analytical solution of such random logistic equations are rarely known. Therefore one has to resort to numerical solution of SDEs for studying various aspects like the time–development of growth patterns, exit frequencies, mean passage times and impact of fluctuating growth parameters. We present some basic aspects of adequate numerical analysis of these random extensions of these models such as numerical regularity and mean square convergence. The problem of keeping reasonable boundaries for analytic solutions under discretization plays an essential role for practically meaningful models, in particular the preservation of intervals with reflecting or absorbing barriers. A discretization of the continuous state space can be circumvented by appropriate methods. Balanced implicit methods (see Schurz, IJNAM 2 (2), p. 197–220, 2005) are used to construct strongly converging approximations with the desired monotone properties. Numerical studies can bring out salient features of the stochastic logistic models (e.g. almost sure monotonicity, almost sure uniform boundedness, delayed initial evolution or earlier points of inflection compared to deterministic model).

Key words. Logistic growth, stochastic logistic equation, properties of solutions, numerical methods, balanced implicit methods, boundedness, convergence, stability, monotonicity.