Analysis and Discretization of Semi-Linear Stochastic Wave Equations with Cubic Nonlinearity and Additive Space-Time Noise

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ABSTRACT: One-dimensional wave equations with cubic power law perturbed by Q-regular additive space-time random noise are considered. These models describe the displacement of noisy nonlinear strings excited by state-independent random external forces. The presented analysis is based on the representation of its solution in form of Fourier-series expansions along the eigenfunctions of Laplace operator with random Fourier coefficients (the so-called commutative case). We shall discuss existence and uniqueness of (strong) solutions using energy-type methods based on the construction of Lyapunov-functions. Appropriate truncations and finite-dimensional approximations are presented while using an approach exploiting the explicit knowledge on eigenfunctions of related second order differential operators. Moreover, some nonstandard partial-implicit difference methods for their numerical integration are suggested in order to control its energy functional in a dynamically consistent fashion.

Key words and phrases. Semilinear stochastic wave equations, cubic nonlinearity, space-time noise, white noise, additive noise, energy identity, Lyapunov functionals, truncation technique, partial-implicit numerical method, discretization of energy, Fourier expansions.