

# Analysis of Noise-Induced Transitions for Hopf System with Additive and Multiplicative Random Disturbances

Preprint m-06-005

Irina Bashkirtseva\*, Lev Ryashko\*, and Henri Schurz†

\**Department of Mathematical Physics, Ural State University, 620083, Ekaterinburg, Russia*

†*Department of Mathematics, Mail code 4408, Southern Illinois University Carbondale, 1245 Lincoln Drive, Carbondale, IL 62901, USA*

**ABSTRACT:** An analysis of the classical Hopf differential system perturbed by multiplicative and additive noises is carried out. An explicit representation for the stationary probability density function is found as an analytical solution of related Fokker-Planck equation. The difference in the response of Hopf systems perturbed by additive and multiplicative random noises is investigated. That difference can be seen in the zone of the transition from the trivial equilibrium point to noisy limit cycle. A delaying shift of the Hopf bifurcation point induced by multiplicative noise is recognized. In fact, an explicit formula of the radius of the stochastic limit cycle as a function of the involved parameters is stated. The phenomenon of inverse stochastic bifurcation in which auto-oscillations are suppressed by multiplicative noise is clearly observed. Eventually, the analytical description of the probability density of the randomly forced Hopf system offers the excellent possibility to test and compare the accuracy of different numerical schemes with respect to the replication of stochastic limit cycles. The superiority of the linear-implicit trapezoidal-type method is demonstrated in this respect.