Nonlinear Stochastic Wave Equations in $\mathbb{R}^1$ with Power-Law Nonlinearity and Additive Space-Time Noise

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ABSTRACT:
This paper is devoted to the qualitative study of properties of one-dimensional stochastic wave equations perturbed by power-law-type nonlinearities and additive space-time random noise. The noise with an Fourier expansion along the eigenfunctions of the Laplace operator is supposed to be Q-regular, white in time and generally spatially correlated. We focus on the behavior of the generalized energy functional incorporating also the energy part which is due to the viscous damping force both in continuous and discrete time. In continuous time, we suggest to exploit the technique of Fourier expansions and appropriate truncation by finite-dimensional systems controlled by the generalized energy functional. It is shown that the mean energy grows linearly in time (with or without damping) and is governed by a kind of trace formula. In discrete time, we suggest to take partial-implicit midpoint-type methods for its adequate discretization. These nonstandard semi-analytical numerical methods possess the property of conserving the generalized expected energy with random initial data. Moreover, we can apply our techniques to estimate some probabilities of large fluctuations of functionals of its solutions.