An Oscillation Theorem for 2nd Order Stochastic Differential Equations and Stochastic Oscillators with Additive Noise

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ABSTRACT: Consider 2nd order stochastic differntial equations (SDEs)

$$\ddot{x} + \omega^2 x = \sigma\xi(t)$$

perturbed by Gaussian white noise ξ, where $\omega \neq 0$ and $\sigma \neq 0$ are any real parameters. We prove that the solution x and its derivative $\dot{x}$ have infinitely many zeros (almost surely) for all adapted initial values $x(0)$, $\dot{x}(0)$. That is, that all components of the related continuous time stochastic system have sustained oscillations around 0 as time t tends to + infinity.

Key words and phrases. Random ODEs; Stochastic Oscillators; Undamped Oscillator; Random Oscillations; Stochastic Differential Equations; Additive White Noise; quasi-Periodic Solutions; Qualitative Behavior; Asymptotic Behavior