

New Stochastic Integrals, Oscillation Theorems and Energy Identities

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ABSTRACT: This paper is divided into three parts on diverse aspects of stochastic analysis, stochastic integration and 2nd order SDEs. We report on new results concerning

- 1) newly defined stochastic integrals such as the stochastic Simpson integral and its relation to the recently introduced stochastic α -integral by the author (DSA, Vol. 15 (2), 2006),
- 2) oscillation theorems for second order stochastic differential equations (SDEs) which show the almost sure oscillation property of all linear undamped oscillators perturbed by additive, non-degenerate martingale-type noise for all measurable random initial data (this generalizes results from X. Mao (1997) and Markus and Weerasinghe (1988)),
- 3) expected energy formulas for linear stochastic oscillators with additive noise under adequate discretization by midpoint-type methods (the latter generalizes independent results from Hong, Scherer and Wang (NPSC, Vol. 14 (1), 2006) and the author (preprints, SIU, 2004, 2005)). New energy-exact numerical methods are introduced for the first time.

These results can be applied to quadrature methods such as Newton-Cotes formulas for stochastic integrals, to analysis of the oscillatory and energy behavior of stochastically perturbed Schrödinger equations, stochastic oscillators, beam models and stochastic wave equations for randomly vibrating strings.

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