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**MATH 150 SKILLS ASSESSMENT**

The purpose of this test is purely diagnostic (before beginning your review, it will be helpful to assess both strengths and weaknesses). All of the test problems are essential to first semester calculus. Answers are provided, and each answer has references to the relevant review topics. If anything is unclear, the review material should help.

1. Sketch a graph – indicate the domain and range of  $f$ .

a)  $f(x) = x^2 + 2$

b)  $f(x) = |x - 2|$

c)  $f(x) = -\ln x$

2. Given  $f(x) = \begin{cases} x^2, & -3 \leq x < 0 \\ x + 1, & 0 \leq x < 2 \\ 4, & 2 \leq x \leq 5 \end{cases}$

a) Find  $f(-3)$ ,  $f(0)$ ,  $f(2)$ ,  $f(e)$ .

b) Sketch a graph of  $f$ .

3. a) For the function  $f(x) = x^2 - 2x$ , find and simplify  $f(x + h) - f(x)$ .

b) Let  $f(x) = 2x + 1$ ,  $g(x) = \sqrt{x}$ ,  $h(x) = \sin x$ . Find the following compositions:

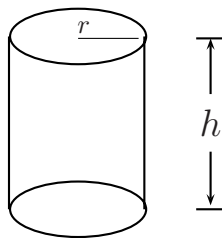
i)  $f(g(x))$

ii)  $h(g(x))$

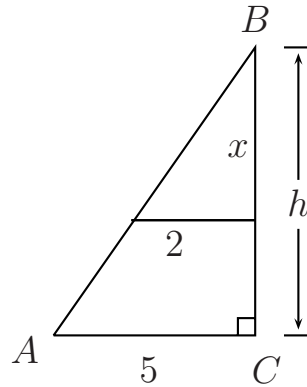
iii)  $g(h(f(x)))$

3. c) Find and simplify the difference quotient  $\frac{f(x) - f(3)}{x - 3}$  if  $f(x) = \frac{1}{x}$ .
- d) Find the difference quotient  $\frac{f(x+h) - f(x)}{h}$  given  $f(x) = \sqrt{x+2}$ . Rationalize the numerator.
4. a) Simplify  $\frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2}$ .
- b) Change  $x^{1/3} + (x-1) \cdot \frac{1}{3}x^{-2/3}$  to an equivalent form whose denominator is  $3x^{2/3}$ .
- c) Given  $f(x) = \frac{-5x(x-4)}{2\sqrt{5-x}}$ ;
- find the domain of  $f$ .
  - find the intervals where  $f > 0$  and  $f < 0$ .
5. a) Given  $y = \ln(x\sqrt{x^2+1})$ , convert into an expression involving sums, differences, and multiples of logarithms.
- b) Simplify:  $e^{-2\ln x}$
- c) Solve for  $x$ :
- $\ln(x-1) = 2$ .
  - $e^{2x} - 2xe^{2x} = 0$

6. a) Given  $g(x) = \frac{2x - 3}{x^2 + 1}$ ,
- state the domain of  $g$ ;
  - find  $g(0)$ ;
  - find all  $x$  such that  $g(x) = 0$ ;
  - find all asymptotes of  $g$ .
- b) Sketch a graph of  $f(x) = (x^2 - 3)(5 - x)^2$ ; include all intercepts.
- c) Sketch a graph of  $f(x) = \frac{x + 3}{3 - 2x}$ ; include asymptotes and all intercepts.
7. a) The position of an object above the ground is given by  $s(t) = 128t - 16t^2$ , with  $s$  measured in feet and  $t$  in seconds.
- Find the maximum height reached by the object.
  - How long does it take for the object to return to the ground?
- b) A right circular cylinder with radius  $r$  and height  $h$  has a volume of  $5 \text{ cm}^3$ . Express its surface area as a function of  $r$ .



7. c) Express the area of  $\triangle ABC$  as a function of  $x$ .

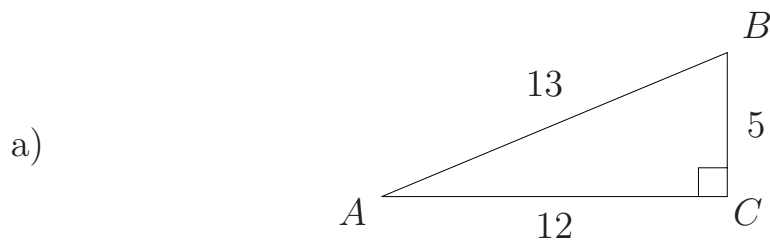


8. a)  $360^\circ = \underline{\hspace{2cm}}$  radians.

b)  $\underline{\hspace{2cm}}^\circ = \frac{5\pi}{6}$  radians.

- c)  $225^\circ = \underline{\hspace{2cm}}$  radians.

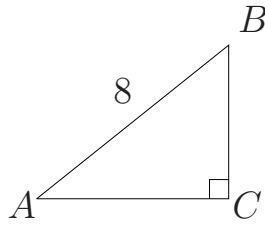
9. For each figure below, find exact values for the quantities indicated.



$\sin A = \underline{\hspace{2cm}};$        $\sin B = \underline{\hspace{2cm}};$        $\cos A = \underline{\hspace{2cm}};$

$\tan A = \underline{\hspace{2cm}};$        $\sec B = \underline{\hspace{2cm}};$        $\cot A = \underline{\hspace{2cm}}.$

9. b)



If angle  $A = \frac{\pi}{6}$ , then side  $BC =$ \_\_\_\_\_.

If angle  $A = \frac{\pi}{4}$ , then side  $AC =$ \_\_\_\_\_.

10. Consider a circle of radius 2 centered at the origin. A central angle  $\theta = \frac{\pi}{3}$  radians will subtend (“mark off”) an arc on the circle of length \_\_\_\_\_.

11. a) Suppose  $0 < \theta < \frac{\pi}{2}$ . If  $\sin \theta = x$ , find  $\cos \theta$  and  $\tan \theta$  in terms of  $x$ .

b) In which quadrant does  $\theta$  terminate if  $\cos \theta < 0$  and  $\tan \theta < 0$ ?

12. Without the use of a calculator, find the exact value of each expression below.

a)  $\sin \frac{7\pi}{6}$

b)  $\cos \left( \frac{-\pi}{2} \right)$

c)  $\tan \frac{7\pi}{4}$

d)  $\sec \frac{13\pi}{6}$

e)  $\sin \left( \frac{-15\pi}{4} \right)$

13. Without using a calculator, complete the table below giving exact values.

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \theta$											
$\cos \theta$											
$\tan \theta$											

14. By memory, graph  $y = \cos x$ ,  $-2\pi \leq x \leq 2\pi$ . Label all intercepts, maxima, and minima.

15. Verify the following trigonometric identities.

a)  $\frac{1 + \sec \theta}{\csc \theta} = \sin \theta + \tan \theta$

b)  $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1$

c)  $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$

16. Evaluate the following without a calculator.

a)  $\arcsin(1)$  or  $\sin^{-1}(1)$

b)  $\arccos(0)$

c)  $\arcsin\left(-\frac{1}{2}\right)$

d)  $\tan(\arcsin(x))$ , where  $0 < x < 1$

17. Without using a calculator, graph  $f(x) = \arctan x$  (or  $f(x) = \tan^{-1} x$ ).

Domain  $f =$  \_\_\_\_\_.

Range  $f =$  \_\_\_\_\_.

18. Solve the following trigonometric equations.

a)  $\cos \theta = -\frac{\sqrt{3}}{2}$

b)  $2 \sin^2 \theta - \sin \theta - 1 = 0, 0 \leq \theta < 2\pi$

c)  $\sin \theta + \sin 2\theta = 0, 0 \leq \theta < 2\pi$

d)  $\sin \theta - \cos \theta = 0, 0 \leq \theta < 2\pi$