Meeting: 1011, Lincoln, Nebraska, SS 17A, Special Session on Calculus of Variations

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It is still not known if the radial cavitating minimizers obtained by Ball [Phil. Trans. R. Soc. Lond. A 306 (1982), 557–611] (and subsequently by many others) are global minimizers of any physically reasonable energy

$$E(\mathbf{u}) = \int_B W(\nabla \mathbf{u}(\mathbf{x}))\, d\mathbf{x}$$

($B \subset \mathbb{R}^3$ is the unit ball). We thus consider the related problem of obtaining necessary conditions for radial solutions to be minimizers with respect to non-radial perturbations. A standard blowup argument applied to either an inner or an outer variation yields a new inequality that has yet to be verified for most W. However, in the special case when $W(\mathbf{F}) = \frac{\mu}{2}|\mathbf{F}|^2 + h(\det \mathbf{F})$ we show that the inequality produced by an inner variation is a well-known inequality, first proven by Brezis, Coron, and Lieb [Comm. Math. Phys. 107 (1986), 649–705] and which arises in the theory of nematic liquid crystals:

$$\int_B |\nabla \mathbf{n}(\mathbf{x})|^2\, d\mathbf{x} \geq \int_B \left|\nabla\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right)\right|^2 d\mathbf{x} = 8\pi$$

for all $\mathbf{n} \in W^{1,2}(B, \partial B)$ that satisfy $\mathbf{n}(\mathbf{x}) = \mathbf{x}/|\mathbf{x}|$ on ∂B. (Received July 29, 2005)