

The convergence of regularised minimisers for cavitation problems in nonlinear elasticity

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ABSTRACT. Consider a nonlinearly elastic body which occupies the region $\Omega \subset \mathbb{R}^m$ in its reference state and which is held in tension under prescribed boundary displacements on $\partial\Omega$. Let $\mathbf{x}_0 \in \Omega$ be any fixed point in the body. It is known from variational arguments that, for sufficiently large boundary displacements, there may exist discontinuous weak solutions of the equilibrium equations corresponding to a hole forming at \mathbf{x}_0 in the deformed body (this is the phenomenon of cavitation). For each $\epsilon > 0$, define the regularised domains $\Omega_\epsilon = \Omega \setminus \overline{B_\epsilon(\mathbf{x}_0)}$ which contain a pre-existing hole of radius $\epsilon > 0$ centred on \mathbf{x}_0 . Now consider the corresponding mixed displacement/traction problem on Ω_ϵ in which the boundary $\partial\Omega$ is subject to the same boundary displacements and the deformed cavity surface (i.e. the image of ∂B_ϵ) is required to be stress free. It follows from variational arguments that there exists a weak solution \mathbf{u}_ϵ of this problem for each $\epsilon > 0$. In this paper we prove convergence of these regularised minimisers \mathbf{u}_ϵ in the limit as $\epsilon \rightarrow 0$. In particular, we show that if $\epsilon_n \searrow 0$ then, passing to a subsequence, $\mathbf{u}_{\epsilon_n} \rightarrow \mathbf{u}$ where \mathbf{u} is a minimiser for the original pure displacement problem on Ω . Finally, we study the effect on cavitation of regularising the variational problem by introducing a surface energy term which penalises the formation and growth of cavities.

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