Math 405 Sample Final

Sections 1

Instructions:

1. Write your name on this answer booklet.
2. Read each question carefully.
3. Please write legibly.
4. TO ENSURE FULL CREDIT, EXPLAIN YOUR WORK FULLY.
5. This exam has 8 pages.
6. The total number of points on this exam is 100.
7. Books and notes are not allowed in this exam.
8. Independent work is expected.

Name: _____

1. _____
2. _____
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5. _____
6. _____
7. _____

Total:

1. Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix} \mathbf{x}$$

2. Find the general solution of the given system. Sketch a phase portait of the system. Find the solution satisfying the initial condition: $\mathbf{x}(0) = (1, 1)^T$, and describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \mathbf{x}.$$

3. Find the general solution of the given system. Sketch a phase portait of the system. Find the solution satisfying the initial condition: $\mathbf{x}(0) = (2, 3)^T$, and describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ 3 & 7 \end{pmatrix} \mathbf{x}, \qquad .$$

4. Find the general solution of the given system (using variation of parameters):

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}.$$

5. Find the critical points for the system

$$\frac{dx}{dt} = x + x^2 + y^2, \qquad \frac{dy}{dt} = y - xy,$$

and discuss the type and stability of these critical points. (You should review all similar problems of the homeworks)

6. Consider the following system

$$\frac{dx}{dt} = -x^3 + xy^2, \qquad \frac{dy}{dt} = -2x^2y - y^3.$$

Use a Liapunov function to show the origin is an asymptotically stable critical point. (You should review all similar problems of the homeworks)

7. (a) Show that the system

$$\frac{dx}{dt} = -y + xf(r)/r, \qquad \frac{dy}{dt} = x + yf(r)/r$$

has periodic solutions corresponding to the zeros of $f(r)$. What is the direction of motion on the closed trajectories in the phase plane?

(b) Let $f(r) = r(r-2)^2(r^2-4r+3)$. Determine all periodic solutions and determine their stability characteristics.

8. The following is expressed in polar coordinates. Determine all periodic solutions, all limit cyccles and determine their stability characteristics.

$$\frac{dr}{dt} = \sin \pi r, \qquad \frac{d\theta}{dt} = 1.$$